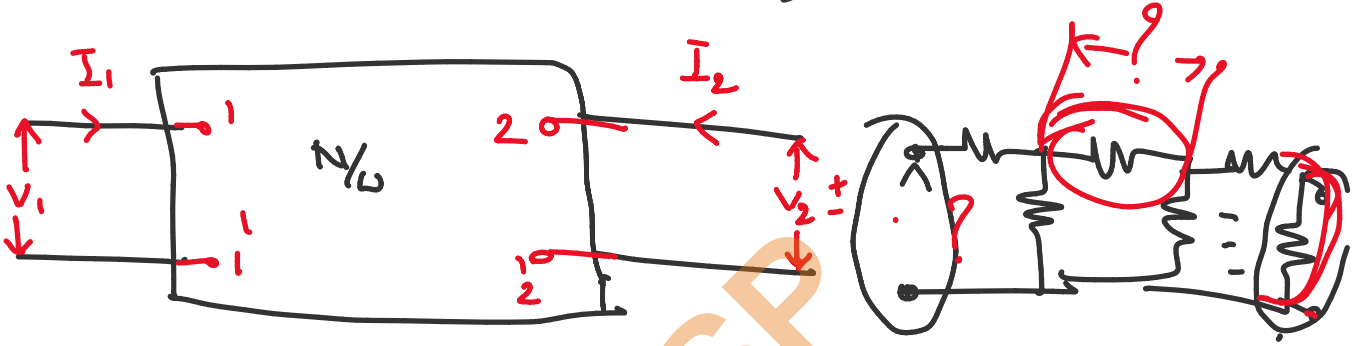
Introduction

18 December 2020 09:27

Two port network

↓

(A connection of many elements to perform some meaningful work)



A pair of terminals thro' which the signal enters or leaves the n/w is called a port.

$\left.\frac{V_1}{I_2}\right|$ or $\left.\frac{I_2}{V_1}\right|$ ___ .

To be represented acc. to some convenience of us?

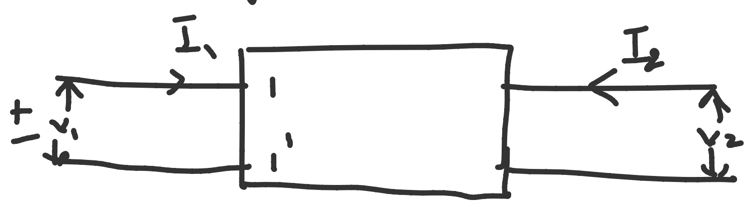
① Series – Impedance ✓ (Two port

② Parallel – Admittance

② Parallel - Admittance (Two port N/w parameters)

③ Cascade - ABCD

④ Amplifier - h - parameters



V_1, I_1, V_2, I_2 are my parameters of interest

(2 parameters to be dependent & another 2 parameters to be independent)

Active port - Source

Passive port - No source

Two port N/w parameters - All parameters are obtained conditionally

ex:- $\left.\frac{V_1}{I_1}\right|_{I_2=0}$ $\left.\frac{V_1}{V_2}\right|_{I_1=0}$.

Not Generalized Ratio. × Conditional Ratio

Generalized - N/w Function

Z-Parameters (open circuit z parameters)

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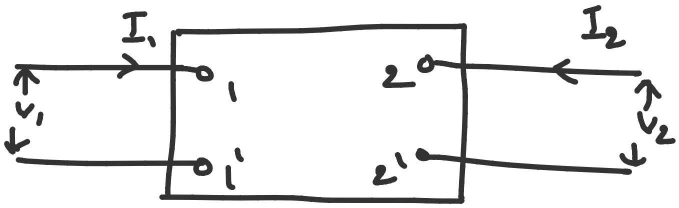


V_1, I_1, V_2, I_2

Z-parameters

I_1, I_2 - Independent parameters

V_1, V_2 - Dependent parameters

$$V_1 = f(I_1, I_2)$$
$$V_2 = f(I_1, I_2)$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{— (1)}$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{— (2)}$$

Z_{12} → Port no; of the independent parameter

Port no; of the parameter to be calculated

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$ — Open ckt input impedance

— Driving point Open ckted impedance

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$ — Transfer Impedance at port 1 with port 2 o.c.

— (Open ckt Transfer Impedance)

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$ — Transfer impedance at port 2 with port 1 o.c.

$$Z_{22} = \left.\frac{V_2}{}\right|$$ — Driving pt impedance at port 2 with port 1 o.c.

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$ — Driving pt impedance at port 2 with port 1 o.c. (Open ckt o/p impedance)

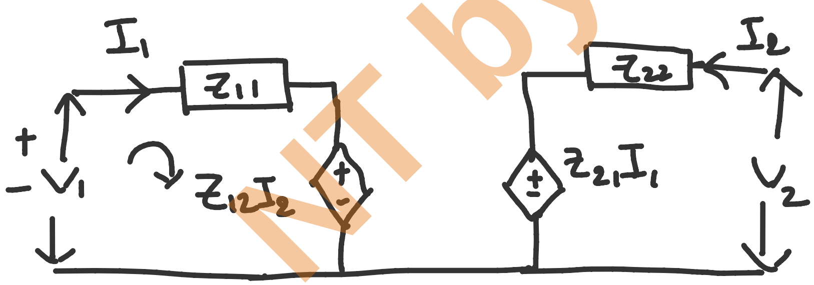
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Rightarrow V = Z I$$

V → Voltage Matrix; Z → Impedance Matrix; I → Current matrix

$$\left.\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \quad \text{— (1)} \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \quad \text{— (2)} \end{aligned}\right\} \text{Mesh eqtns}$$



— Pictorial or ckt of the two port n/w eqtns

$Z_{21} = Z_{12}$ — Reciprocal n/w or Bilateral n/w

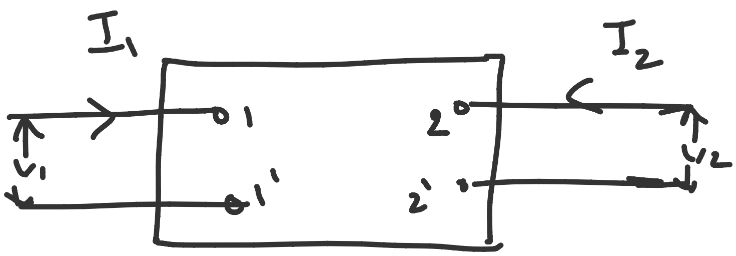
$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

— All parameters are in 'Ω'

16 January 2021 14:59

Y-Parameters (Short Circuited Y Parameters)

18 December 2020 10:01



I_1, I_2 - dependent

V_1, V_2 - Independent.

I in terms of voltages.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$ - Driving pt admittance at port 1 with port 2 s.c

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

↳ Admittance

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$ Transfer Admittance at port 1 with port 2 S.C

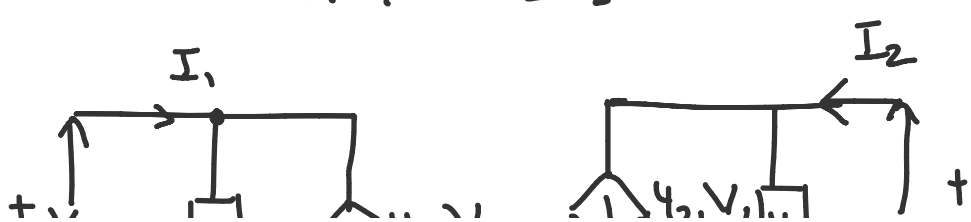
$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$ - Trans. Admittance at port 2 with port 1 S.C.

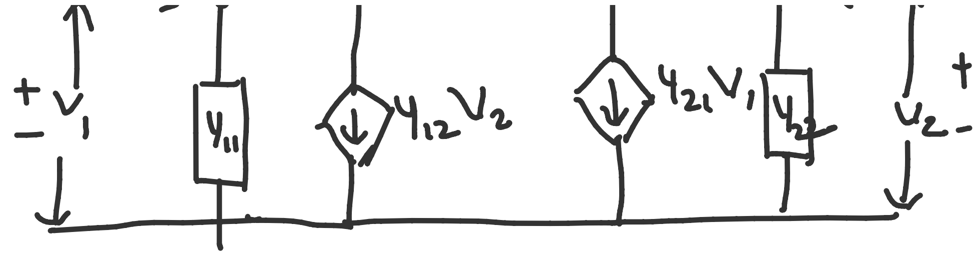
$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$ - Driving pt Admittance at port 2 with port 1 S.C.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Dependent current source





All parameter are is $\underline{\mho}$

ABCD Parameters(Transmission Parameters)

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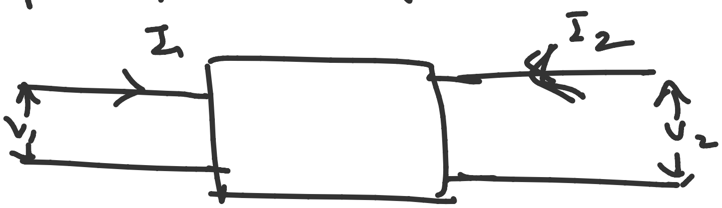
Input port is exp. in terms of o/p port parameters



V_2, I_2 are independent
V_1, I_1 are dependent

$$V_1 = AV_2 + B(-I_2)$$

$$I_1 = CV_2 + D(-I_2)$$

To make the current going out of the o/p positive a − sign is attributed to I_2 incoming Current

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

B & D

$-I_2$

port 1

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Port 2

$A = \left. \dfrac{V_1}{V_2} \right|_{I_2 = 0}$ — Open ckt the o/p port

– Gain parameter
No units

$C = \left. \dfrac{I_1}{V_2} \right|_{I_2 = 0}$

O.C. Admittance parameter

$\dfrac{1}{C} = \left. \dfrac{V_2}{I_1} \right|_{I_2=0} = Z_{21}$

$\boxed{Z_{21} = 1/C}$

$I_1 \; I_2 = 0$

$$\boxed{Z_{21} = \frac{1}{C}}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \Rightarrow -B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{1}{Y_{21}}$$

Impedance pa-
with port 2 S.C

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} \Rightarrow -D = \left.\frac{I_1}{I_2}\right|_{V_2=0} -$$

Current gain no
units

Inverse Transmission Parameters

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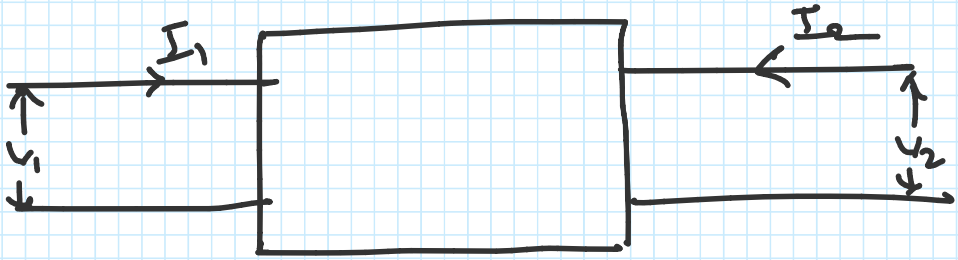


O/p por parameters are rep. in terms of I/p por parameters

$$V_2 = A'V_1 - B'I_1$$

$$I_2 = C'V_1 - D'I_1$$

$$A' = \frac{V_2}{V_1}\bigg|_{I_1=0}$$

$$C' = \frac{I_2}{V_1}\bigg|_{I_1=0}$$

$$-B' = \frac{V_2}{I_1}\bigg|_{V_1=0}$$

$$-D' = \frac{I_2}{I_1}\bigg|_{V_1=0}$$

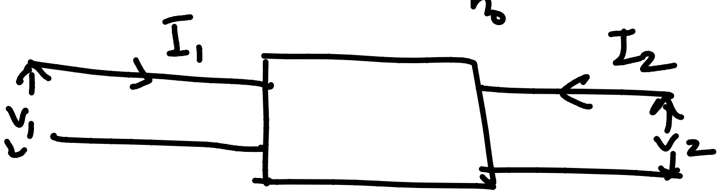
hybrid parameters(h-parameters)

18 December 2020 22:05

hybrid parameters — used in analysis of Amplifiers.

→ A_v, A_I, R_i, R_o → h-parameters exactly.

$\downarrow$ $1/h_o$



V_1, I_2 — Dependent Parameters

I_1, V_2 — Independent

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{— (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{— (2)}$$

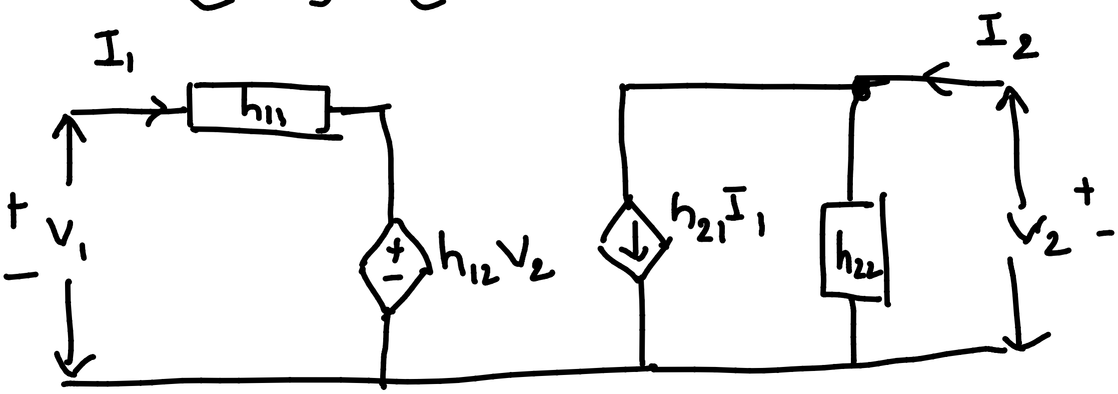
$$h_i = h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$ Port 2 S.C — Input impedance at Port 1 (Ω)

$$h_r = h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$ Port 1 O.C — Reverse Voltage Gain (No units)

$$h_f = h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$ Port 2 S.C — Forward Current gain (No units)

$$h_o = h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$ Port 1 O.C — O/P Admittance ($\mho$)

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$



Inverse hybrid parameters

19 December 2020 10:44

I_1, V_2 are expressed in terms of V_1, I_2

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$ O.C I/p Admittance

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$ I/p shorted, Reverse current gain

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$ O/p O.C, f. Voltage Gain

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$ – I/p shorted, O/p impedance

Represent one parameter in terms of the other parameters

19 December 2020 10:47

Rep of Y in terms Z.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V = Z I \quad — ①$$

from eq ①

$$I = V/Z$$

$$I = V Z^{-1} — ①'$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I = Y V \quad — ②$$

From eq ①' & ②

$$Y = Z^{-1}.$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$Y = Z^{-1} = \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix} / \Delta Z$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{22}/\Delta Z & -Z_{12}/\Delta Z \\ -Z_{21}/\Delta Z & Z_{11}/\Delta Z \end{bmatrix}$$

$$V = Z I \qquad I = Y \cancel{Z} V$$

Z parameters in terms of Y parameters

Z parameters in terms of Y parameters

$$V = I/Y = I Y^{-1}$$

$$Z = Y^{-1} \Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix} \Big/ \Delta Y$$

$$\begin{bmatrix} Z_{11} = Y_{22}/\Delta Y & Z_{12} = -Y_{12}/\Delta Y \\ Z_{21} = -Y_{21}/\Delta Y & Z_{22} = +Y_{11}/\Delta Y \end{bmatrix}$$

$$\boxed{Z = Y^{-1} \text{ or } Y = Z^{-1}}$$

* $Z_{12} = Z_{21} \Rightarrow \dfrac{-Y_{12}}{\Delta Y} = \dfrac{-Y_{21}}{\Delta Y} \Rightarrow \boxed{Y_{12} = Y_{21}}$ *

Rep. of h-parameters in terms of Z- parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad -(1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad -(2)$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad -(3)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -(4)$$

$$h_{11} = \frac{V_1}{I_1}\Bigg|_{V_2 = 0}$$

In eq (2) if $V_2 = 0$

$$0 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{21} I_1 = -Z_{22} I_2$$

Sub (5) in (1)

$$Z_{21} I_1 = -Z_{22} I_2$$

$$I_2 = -\frac{Z_{21}}{Z_{22}} I_1 \quad - (5)$$

Sub (5) in (1)

$$V_1 = Z_{11} I_1 - \frac{Z_{12} Z_{21} I_1}{Z_{22}} =$$

$$V_1 = \frac{(Z_{11} Z_{22} - Z_{12} Z_{21})}{Z_{22}} I_1 \Rightarrow h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = \frac{\Delta Z}{Z_{22}} \quad - (6)$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = -\frac{Z_{21}}{Z_{22}} \quad - (7)$$

Make $I_1 = 0$ in eq (1) & (2)

$$V_1 = Z_{12} I_2$$

$$V_2 = Z_{22} I_2$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = \frac{Z_{12} I_2}{Z_{22} I_2} = \frac{Z_{12}}{Z_{22}}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}} \quad - (8)$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{1}{Z_{22}} \quad - (9)$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \Delta Z / Z_{22} & Z_{12}/Z_{22} \\ -Z_{21}/Z_{22} & 1/Z_{22} \end{bmatrix}$$

ABCD in terms of Z

$V_1 = Z_{11}I_1 + Z_{12}I_2$ — (1) $\quad V_1 = AV_2 - BI_2$ — (3)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ — (2) $\quad I_1 = CV_2 - DI_2$ — (4)

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} = \frac{Z_{11}I_1}{Z_{21}I_1} = \frac{Z_{11}}{Z_{21}} \quad \text{— (5)}$$

$V_1 = Z_{11}I_1$

$V_2 = Z_{21}I_1$

Making $I_2 = 0$ in eq (1) & (2)

$$C = \frac{I_1}{V_2}\Big|_{I_2=0} = \frac{1}{Z_{21}} \quad \text{— (6)}$$

Making $V_2 = 0$ in eq (2)

$$0 = Z_{21}I_1 + Z_{22}I_2$$

$$I_2 = -\frac{Z_{21}I_1}{Z_{22}} \quad \text{— } \cancel{(7)}$$

$$I_1 = -I_2\frac{Z_{22}}{Z_{21}} \quad \text{— (7)}$$

Sub (7) in eq (1)

$$V_1 = Z_{11}\left(-I_2\frac{Z_{22}}{Z_{21}}\right) + Z_{12}I_2$$

$$V_1 = \frac{\left(-Z_{11}Z_{22} + Z_{12}Z_{21}\right)}{Z_{21}}I_2 \quad \text{— (8)}$$

$$B = -\frac{V_1}{I_2} = \frac{\left(-Z_{12}Z_{21} + Z_{11}Z_{22}\right)}{Z_{21}} = \frac{Z_{11}Z_{22} - Z_{21}Z_{12}}{Z_{21}}$$

$$D = -\frac{I_1}{I_2} = \frac{Z_{22}}{Z_{21}}$$

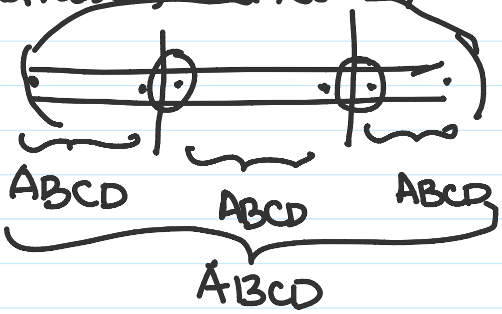
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{Z_{11}}{Z_{21}}, & \dfrac{Z_{11}Z_{22} - Z_{21}Z_{12}}{Z_{21}} \\ \dfrac{1}{Z_{21}} & \dfrac{Z_{22}}{Z_{21}} \end{bmatrix}$$

Cascade Connection, Series Connection, Parallel Connection

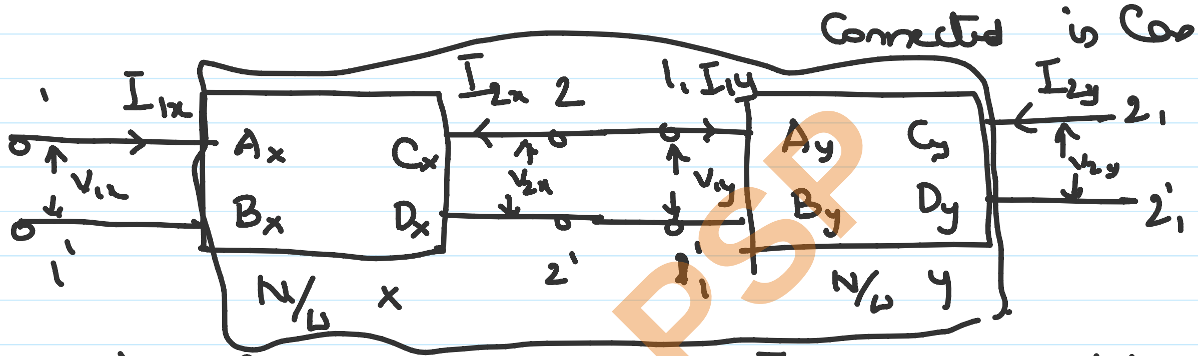
19 December 2020 11:09

Cascade Connection:

Transmission Lines → Cascading



To find the Resultant ABCD parameters of all the Transmission lines Connected in Cascade



V_{2y} & V_{1x} related, I_{2y} & I_{1x} related.

For n/w x

$$\begin{bmatrix} V_{1x} \\ I_{1x} \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} V_{2x} \\ -I_{2x} \end{bmatrix} \quad \text{—} \; (1)$$

For n/w y

$$\begin{bmatrix} V_{1y} \\ I_{1y} \end{bmatrix} = \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_{2y} \\ -I_{2y} \end{bmatrix} \quad \text{—} \; (2)$$

According to Connection

$$\begin{bmatrix} V_{2x} \\ -I_{2x} \end{bmatrix} = \begin{bmatrix} V_{1y} \\ I_{1y} \end{bmatrix} \quad \text{—} \; (3)$$

Writing (2) again

Writing ② again

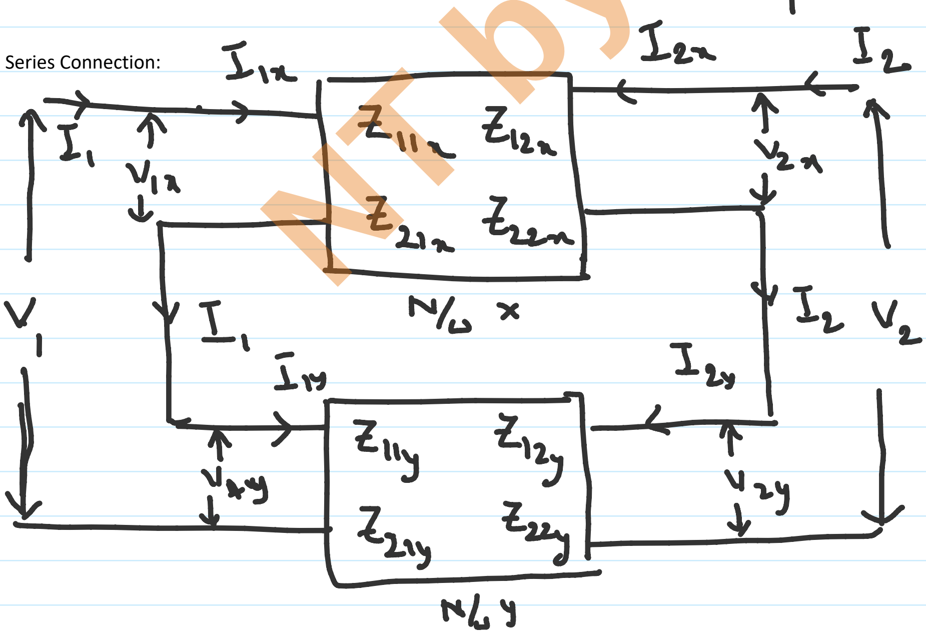
$$\begin{bmatrix} V_{2x} \\ -I_{2x} \end{bmatrix} = \begin{bmatrix} V_{1y} \\ I_{1y} \end{bmatrix} = \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_{2y} \\ -I_{2y} \end{bmatrix} \quad -④$$

Combining ① & ④

$$\begin{bmatrix} V_{1x} \\ I_{1x} \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_{2y} \\ -I_{2y} \end{bmatrix} \quad -⑤$$

The Resultant ABCD parameters of the Cascaded N/w are a Matrix Multiplication of the individual N/w ABCD parameters.

Series Connection:



$I_1 = I_{1x} = I_{1y} \quad -①$ $\qquad V_1 = V_{1x} + V_{1y} \quad -③$

$I_2 = I_{2x} = I_{2y} \quad -②$ $\qquad V_2 = V_{2x} + V_{2y} \quad -④$

For N/w x

$$V_{1x} = Z_{11x} I_{1x} + Z_{12x} I_{2x} \quad -⑤$$

For N/w x

$$V_{1x} = Z_{11x} I_{1x} + Z_{12x} I_{2x} \quad -(5)$$

$$V_{2x} = Z_{21x} I_{1x} + Z_{22x} I_{2x} \quad -(6)$$

For N/w y

$$V_{1y} = Z_{11y} I_{1y} + Z_{12y} I_{2y} \quad -(7)$$

$$V_{2y} = Z_{21y} I_{1y} + Z_{22y} I_{2y} \quad -(8)$$

For the Combined Series n/w ($V_1, V_2 \to I_1, I_2$)

from eq (3), $V_1 = V_{1x} + V_{1y}$ (sub 5 & 7 in 3)

$$V_1 = Z_{11x} I_{1x} + Z_{12x} I_{2x} + Z_{11y} I_{1y} + Z_{12y} I_{2y}$$

(from (1) & (2) eq$^{\underline{n}}$ we know $I_1 = I_{1x} = I_{1y}$, $I_2 = I_{2x} = I_{2y}$)

$$V_1 = (Z_{11x} + Z_{11y}) I_1 + (Z_{12x} + Z_{12y}) I_2 \quad -(9)$$

$|||^{ly}$ from (4) $V_2 = V_{2x} + V_{2y}$ (sub 6 & 8 in 4)

$$V_2 = Z_{21x} I_{1x} + Z_{22x} I_{2x} + Z_{21y} I_{1y} + Z_{22y} I_{2y}$$

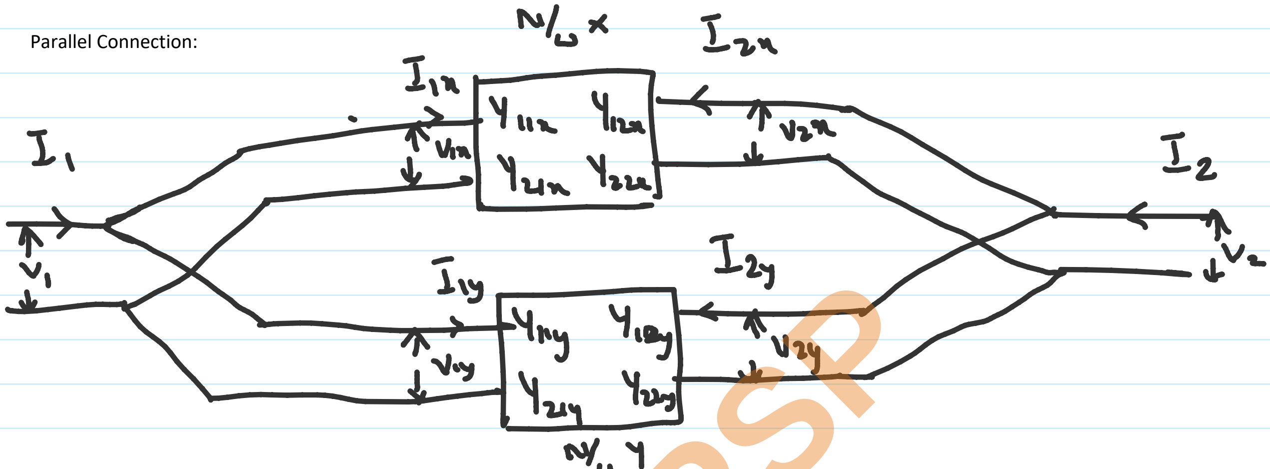
$$V_2 = (Z_{21x} + Z_{21y}) I_1 + (Z_{22x} + Z_{22y}) I_2 \quad -(10)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11x} + Z_{11y} & Z_{12x} + Z_{12y} \\ Z_{21x} + Z_{21y} & Z_{22x} + Z_{22y} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_2 \end{bmatrix} \quad \begin{bmatrix} Z_{21x} + Z_{21y} & \cdots \end{bmatrix}$$

So 2 n/w connected in series, have their Z-parameters are a summation of the Z-parameters of individual n/ws.

Parallel Connection:



$$V_1 = V_{1x} = V_{1y} \quad - (1) \qquad I_1 = I_{1x} + I_{1y} \quad - (3)$$

$$V_2 = V_{2x} = V_{2y} \quad - (2) \qquad I_2 = I_{2x} + I_{2y} \quad - (4)$$

For n/w x

$$I_{1x} = Y_{11x} V_{1x} + Y_{12x} V_{2x} \quad - (5)$$

$$I_{2x} = Y_{21x} V_{1x} + Y_{22x} V_{2x} \quad - (6)$$

For n/w Y

$$I_{1y} = Y_{11y} V_{1y} + Y_{12y} V_{2y} \quad - (7)$$

$$I_{2y} = Y_{21y} V_{1y} + Y_{22y} V_{2y} \quad - (8)$$

from eq (3) $\quad I_1 = I_{1x} + I_{1y}$ (5 & 6 in 7)

$$I_1 = Y_{11x} V_{1x} + Y_{12x} V_{2x} + Y_{11y} V_{1y} + Y_{12y} V_{2y}$$

(from $V_{1x} = V_{1y} = V_1$)

$$I_1 = (Y_{11x} + Y_{11y}) V_1 + (Y_{12x} + Y_{12y}) V_2 \quad - (9)$$

$$I_1 = (Y_{11x} + Y_{11y})V_1 + (Y_{12x} + Y_{12y})V_2 \quad - (9)$$

$$I_2 = Y_{21x}V_{1x} + Y_{22x}V_{2x} + Y_{21y}V_{1y} + Y_{22y}V_{2y}$$

$$I_2 = (Y_{21x} + Y_{21y})V_1 + (Y_{22x} + Y_{22y})V_2 \quad - (10)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11x} + Y_{11y} & Y_{12x} + Y_{12y} \\ Y_{21x} + Y_{21y} & Y_{22x} + Y_{22y} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

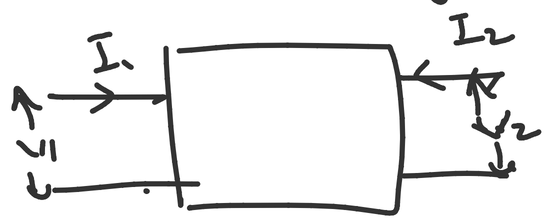
Y parameter of Parallel Connection are summation of Y- parameter of Individual N/w.

Network functions

31 December 2020 10:21

More helpful, more meaningful, more practical

$\frac{V_1}{I_1}, \frac{I_1}{V_1}$



$\frac{V_2}{I_2}, \frac{I_2}{V_2}$

$$G_{21} = \frac{V_2}{V_1}$$

Ratio of parameters of same port is called DP.

— Driving Point Function

Transfer Function — Ratio of parameters b/w different ports then it is transfer

$$\underbrace{\frac{I_1}{I_2}, \frac{I_2}{I_1}}_{\text{Current Ratio}}, \underbrace{\frac{V_1}{V_2}, \frac{V_2}{V_1}}_{\text{Voltage ratio}}, \underbrace{\frac{I_1}{V_2}, \frac{V_2}{I_1}, \frac{I_2}{V_1}, \frac{V_1}{I_2}}_{\text{Trans Imp or Admittance}}$$

$\to$ Two port

z_{21} y_{21} g_{21} z_{21} y_{21}

$Z_{21}(s) \to$ N/w Function

$Y_{21}(s)$ —

$s = -1 \to$ Zero

$s = -3 \to$ Pole

$0 \to$ N/w

$s = \sigma + j\omega$

$$G_{21}(s) = \frac{s^2+3s+2}{s^2+5s+6} = \frac{\boxed{(\cancel{s+2})(s+1)}}{(\cancel{s+2})(s+3)} = \frac{x}{0} = \infty$$

$s = \sigma + j\omega$

$$G_{21}(s) = \frac{\ldots}{s^2+5s+6} = \frac{\ldots}{(s+2)(s+3)} = \infty$$

$s = -2,\ s = -1 \rightarrow$ * Roots of Nr $\rightarrow$ Zeros *

$s = -2,\ s = -3 \rightarrow$ * Roots of Dr $\rightarrow$ Pole *

$$\frac{(s+1)}{(s+3)}$$

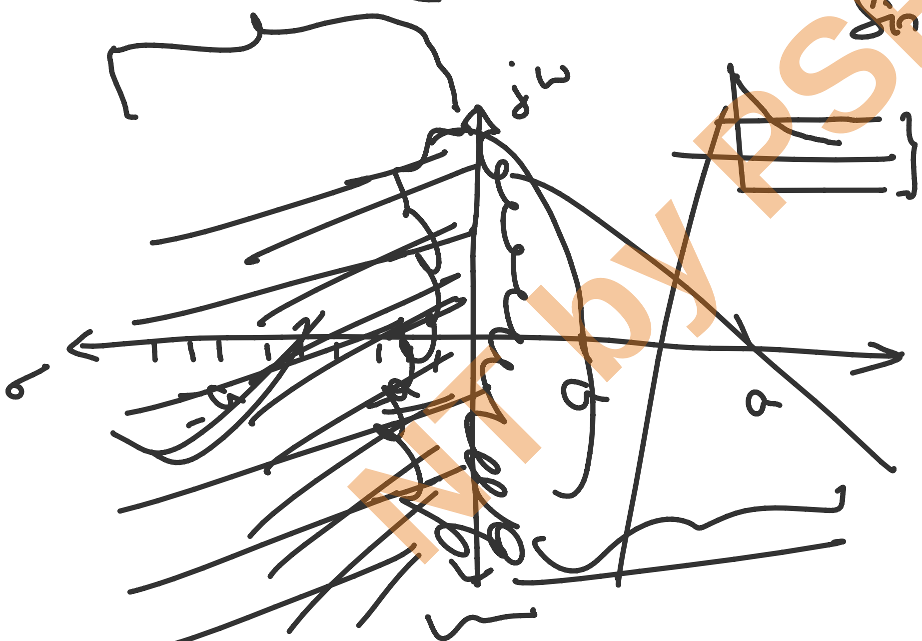
$$G_{21}(s) = \frac{(s-(-2))(s-(-1))}{(s-(-2))(s-(-3))}$$

'∞' → Indeterminant

$(s+1)^2 = s^2+1$ t ✗

$$\frac{(s+1)(s+1)}{}$$

$\sin t$ ✗



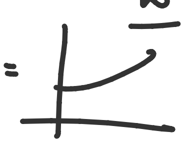
Real part is $\underline{-ve}$

$$\frac{1}{s+a} \qquad e^{-s} \ \ -\infty$$

$$\frac{1}{s-(-a)} \Rightarrow \boxed{e^{-at}}$$

$$\frac{1}{s-a} = \frac{1}{s-(a)} = e^{at} \quad \infty$$

$=$

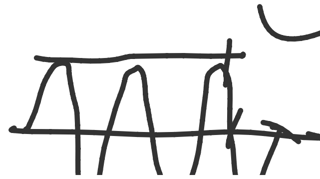


?

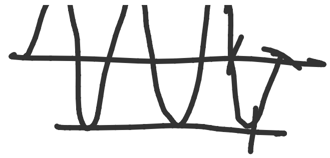
(s^2+a^2) $s = \pm ja$ $(s+ja)\ (s+ja_1)$ ✗

↓

sin or Cos



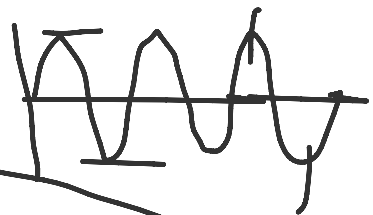
$\underbrace{\sin \text{ or } \cos}$ ✗



$\underbrace{s^2+s+1}_{\text{RLC}} \Rightarrow Ls + \frac{1}{Cs} \Rightarrow \frac{Ls^2+1}{Cs}$

$s^2+a^2 \Rightarrow s = \pm ja$

sin / Cos



$\cancel{(s+ja)(s+ja_1)}$

$as^2+bs+c \rightarrow \underbrace{\text{RLC}}$

$Ls + \frac{1}{Cs} \Rightarrow \frac{(Ls^2+1)}{Cs}$

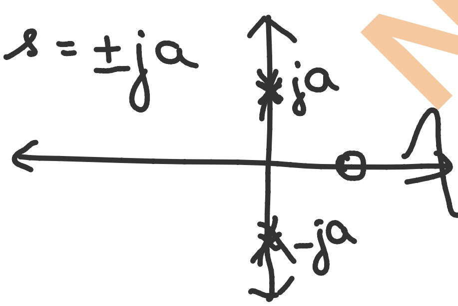
$s^2 = -1/LC \Rightarrow \underbrace{s = \pm j\sqrt{1/LC}}$

$1/s^2 \Rightarrow t$

$\frac{1}{(s^2+a^2)^2} \Rightarrow \underline{t\sin\omega t}$ or $\underline{t\cos\omega t} \rightarrow$ magn

O/P = ∞ ✗

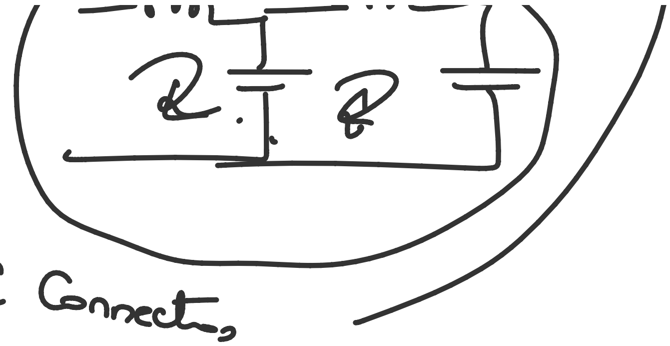
$s = \pm ja$



$$\frac{1}{2s^4+5s^3\underbrace{+3s^2+2s+3}_{\text{RLC}}}$$

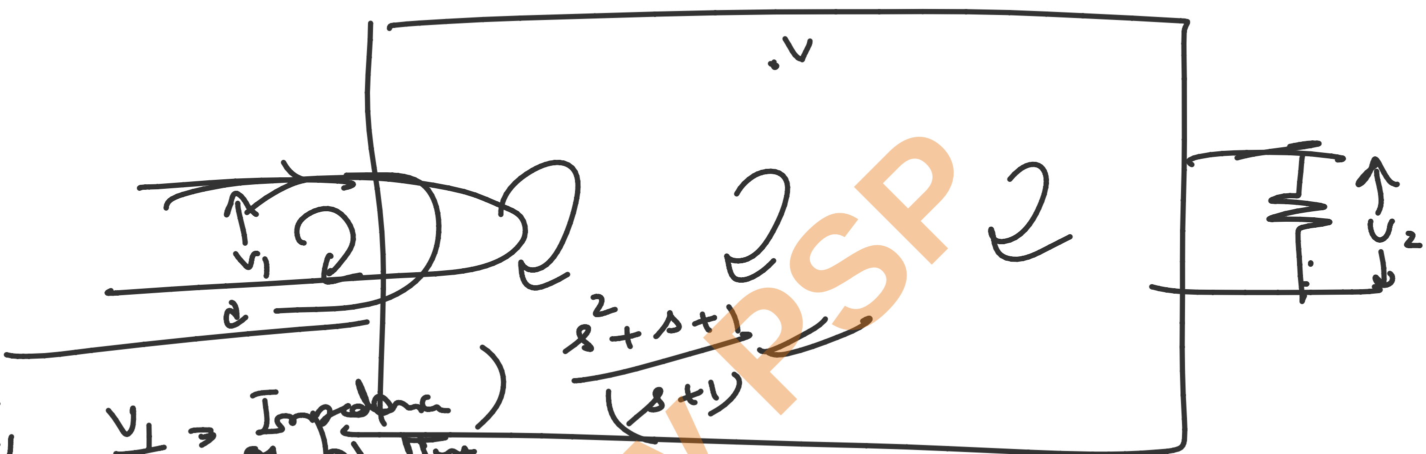
$$\frac{s}{s(\underbrace{s^4+s^2+1})} = \frac{1}{(s^5+\underbrace{s^3}+s)}$$

RLC



$$\frac{1}{s^4+s^2+1} \rightarrow \text{LC Connection}$$

$$\frac{1}{s^4+0+s^2+s+1} \times \rightarrow \text{Cannot forward for a (valid } \gamma_{\omega})$$



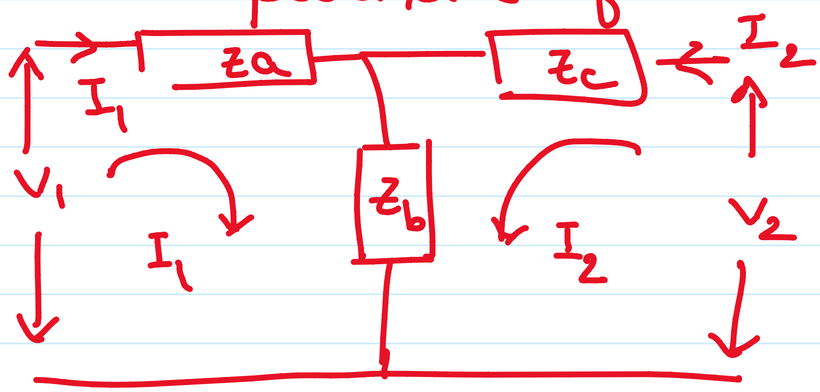
$\frac{I_1}{V_1}$ or $\frac{V_1}{I_1}$ ⇒ Impedance or Admittance

$$G_{21}(s) = \frac{P(s)}{Q(s)} = \frac{V_2}{V_1} = \frac{(+)}{(+\quad)}$$

Problems on two port network parameters

16 January 2021 10:28

① Find the Z-parameters for the ckt given below



$$V_1 = Z_a I_1 + Z_b (I_1 + I_2)$$

$$V_1 = (Z_a + Z_b) I_1 + Z_b I_2 \quad \text{—} (1)$$

$$V_2 = Z_c I_2 + Z_b (I_1 + I_2)$$

$$V_2 = Z_b I_1 + (Z_c + Z_b) I_2 \quad \text{—} (2)$$

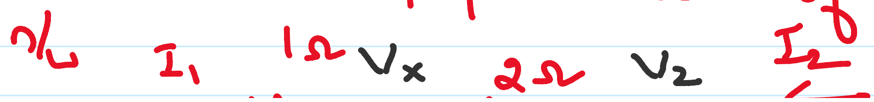
$$\left[\begin{aligned} V_1 &= (Z_a + Z_b) I_1 + Z_b I_2 \\ V_2 &= Z_b I_1 + (Z_c + Z_b) I_2 \end{aligned} \right]$$

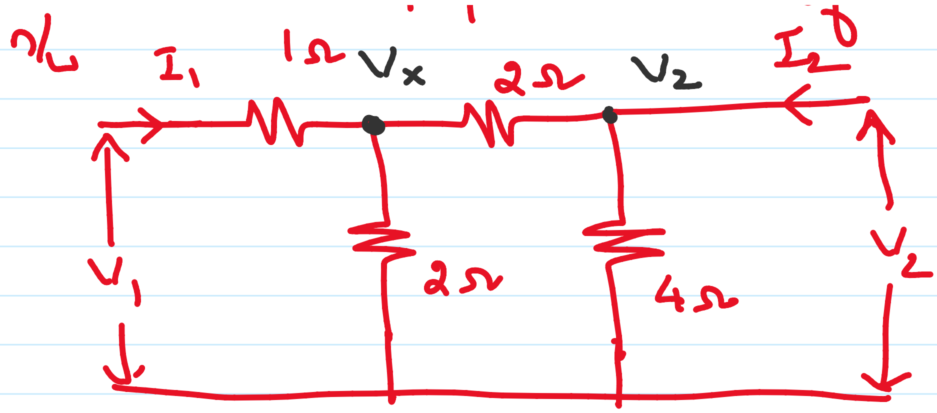
$Z_{11} = Z_a + Z_b$ $\qquad\qquad$ $Z_{21} = Z_b$

$Z_{12} = Z_b$ $\qquad\qquad$ $Z_{22} = (Z_c + Z_b)$

$$\boxed{Z_{12} = Z_{21}}$$ Reciprocal n/w.

② Find the y-parameters of the given n/w





$$I_1 = \frac{V_1 - V_x}{1} \quad - (1)$$

At node V_x
$$\left[\frac{V_x - V_1}{1} + \frac{V_x}{2} + \frac{V_x - V_2}{2} = 0 \quad - (2)\right]$$

At node V_2
$$\frac{V_2 - V_x}{2} + \frac{V_2}{4} = I_2 \quad - (3)$$

Rearranging eq (2) such that V_x is expressed in terms of V_1 & V_2

$$V_x - V_1 + 0.5V_x + 0.5V_x - 0.5V_2 = 0$$

$$2V_x = V_1 + 0.5V_2$$

$$\Rightarrow V_x = \frac{V_1}{2} + \frac{0.5\,V_2}{2}$$

$$V_x = 0.5\,V_1 + 0.25\,V_2 \quad - (4)$$

Sub (4) in (1) & (3)

eq (1) $I_1 = \frac{V_1 - V_x}{1} \Rightarrow I_1 = \frac{V_1 - (0.5V_1 + 0.25V_2)}{1}$

$$I_1 = 0.5V_1 - 0.25V_2 \quad \text{—} (5)$$

eq (3)
$$I_2 = \frac{V_2}{4} + \frac{V_2 - V_x}{2}$$

$$= \frac{V_2}{4} + \frac{V_2 - (0.5V_1 + 0.25V_2)}{2}$$

$$= 0.25V_2 + 0.5V_2 - 0.25V_1 - 0.125V_2$$

$$I_2 = -0.25V_1 + 0.625V_2 \quad \text{—} (6)$$

$$I_1 = 0.5V_1 - 0.25V_2$$
$$I_2 = -0.25V_1 + 0.625V_2$$

$Y_{11} = 0.5\ \mho$, $Y_{12} = -0.25\ \mho$, $Y_{21} = -0.25\ \mho$, $Y_{22} = 0.625\ \mho$

(3) Find the ABCD parameters for the given ckt—

$$V_1 = 1 \times I_1 + 5(I_1 + I_2) \Rightarrow$$

$$V_2 = 2I_2 + 5(I_1 + I_2)$$

$$V_1 = 6I_1 + 5I_2 \quad \text{—} (1)$$

$$V_2 = 5I_1 + 7I_2 \quad \text{—} (2)$$

Making $I_2 = 0$ in eq (1) & (2)

$$V_1 = 6I_1$$
$$V_2 = 5I_1$$

$$\left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{6I_1}{5I_1} = A \Big|_{I_2=0}$$

$$A = 6/5 \quad \text{—} (3)$$

$$V_2 = 5I_1 + 7I_2 \quad -(2)$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$A = 6/5 \quad -(3)$$

$$C\Big|_{I_2=0} = I_1/V_2\Big|_{I_2=0} = 1/5 \quad -(4)$$

Making $V_2 = 0$ in eq (2)

$$5I_1 = -7I_2 \Rightarrow I_1/I_2 = -7/5$$

$$D = -I_1/I_2\Big|_{V_2=0} = 7/5 \quad -(5)$$

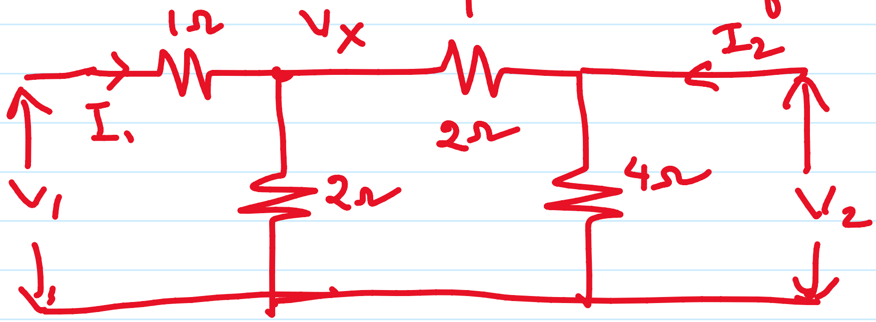
sub $5I_1 = -7I_2$ in eq (1), $B = -\dfrac{V_1}{I_2}\Big|_{V_2=0}$

$$V_1 = 6\cdot\left(-7/5\right)I_2 + 5I_2$$

$$V_1 = \frac{(-42+25)}{5} I_2 = -17/5\, I_2$$

$$B = +17/5 \quad -(6)$$

(4) Find the h-parameters of the n/w.



$$I_1 = \frac{V_1 - V_x}{1} \quad -(1)$$

$$\frac{V_x - V_1}{1} + \frac{V_x}{2} + \frac{V_x - V_2}{2} = 0 \quad -(2)$$

$$\frac{V_2}{4} + \frac{V_2 - V_x}{2} = I_2 \quad -(3)$$

eliminate V_x

$$I_1 = 0.5V_1 - 0.25V_2 \quad -(4)$$

$$\left(I_2 = -0.25V_1 + 0.625V_2 \quad -(5) \right)$$

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

Making $V_2 = 0$ in (4) & (5)

$I_1 = 0.5V_1$

$I_2 = -0.25V_1$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-0.25V_1}{0.5V_1} = \underline{\underline{-0.5}}$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{2I_1}{I_1} = \underline{2\Omega}$$

Making $I_1 = 0$ in eq (4)

$$0 = 0.5V_1 - 0.25V_2$$

$$0.5V_1 = 0.25V_2 \Rightarrow \boxed{V_1 = \frac{V_2}{2}}$$

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} = \frac{1}{2} = 0.5$$

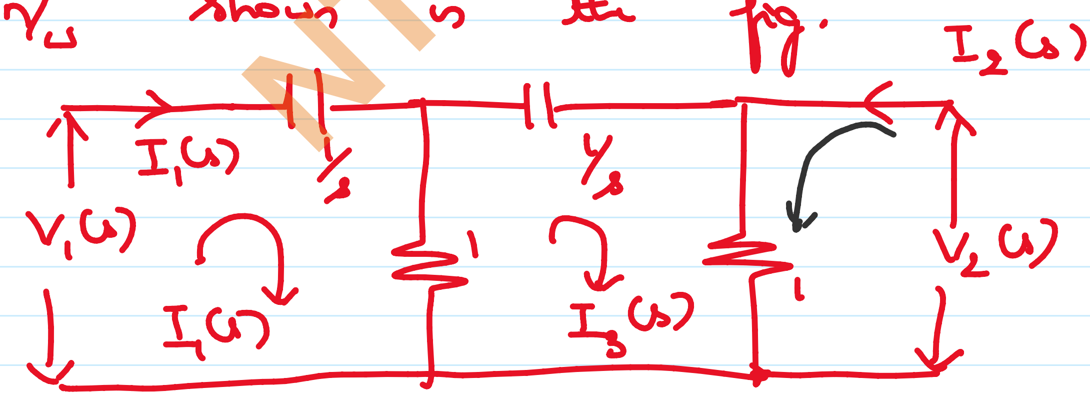
$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0} = 0.5\ \mho$$

$$I_2 = -0.25V_1 + 0.625V_2$$

$$I_2 = -0.25\times\frac{V_2}{2} + 0.625V_2$$

$$I_2 = -0.125V_2 + 0.625V_2 = 0.5V_2$$

⑤ Find the z-parameters of the RC ladder n/w shown in the fig.



$$V_1(s) = \frac{1}{s}I_1(s) + 1\left(I_1(s) - I_3(s)\right) \quad — ①$$

$$1\left(I_3(s) - I_1(s)\right) + \frac{1}{s}I_3(s) + 1\left(I_3(s) + I_2(s)\right) = 0$$

$$V_2(s) = \left(I_3(s) + I_2(s)\right)\cdot 1 \quad — ②$$

$$V_2(s) = \left(I_3(s) + I_2(s)\right) 1 \quad \text{— (3)}$$

rearranging eq (2) to get I_3 in term of I_1 & I_2

$$I_3(s) - I_1(s) + \frac{1}{s} I_3(s) + I_3(s) + I_2(s) = 0$$

$$I_3(s)\left[2 + \frac{1}{s}\right] = -I_2(s) + I_1(s)$$

$$I_3(s)\left[\frac{2s+1}{s}\right] = I_1(s) - I_2(s)$$

$$I_3(s) = \frac{s\left[I_1(s) - I_2(s)\right]}{2s+1} \quad \text{— (4)}$$

sub (4) in (1) & (3)

eq (1) $\quad V_1(s) = \frac{1}{s} I_1(s) + 1\left(I_1(s) - I_3(s)\right)$

$$V_1(s) = \left(1 + \frac{1}{s}\right) I_1(s) - s\left[\frac{I_1(s) - I_2(s)}{2s+1}\right]$$

$$= \left[\left(\frac{s+1}{s}\right) - \frac{s}{(2s+1)}\right] I_1(s) + \frac{s}{2s+1} I_2(s)$$

$V_1(s) \quad \left[(2s+1)(s+1) - s^2\right] I_1(s) + \quad s \quad I_2(s)$

$$V_1(s) = \left[\frac{(2s+1)(s+1) - s^2}{s(2s+1)}\right] I_1(s) + \frac{s}{2s+1} I_2(s)$$

$$= \left[\frac{2s^2+2s+s+1-s^2}{s(2s+1)}\right] I_1(s) + \frac{s}{2s+1} I_2(s)$$

$$V_1(s) = \left(\frac{s^2+3s+1}{s(2s+1)}\right) I_1(s) + \frac{s}{2s+1} I_2(s) \quad \text{--- } 5$$

Z_{11} $\qquad$ Z_{12}

eq 3

$$V_2(s) = \left(I_3(s) + I_2(s)\right) \times 1$$

$$V_2(s) = \frac{s\left(I_1(s) - I_2(s)\right)}{2s+1} + I_2(s)$$

$$V_2(s) = \frac{s\, I_1(s)}{2s+1} + \left[\frac{-s}{2s+1} + 1\right] I_2(s)$$

$$V_2(s) = \frac{s\, I_1(s)}{2s+1} + \frac{s+1}{2s+1} I_2(s)$$

Z_{21} $\qquad$ Z_{22}

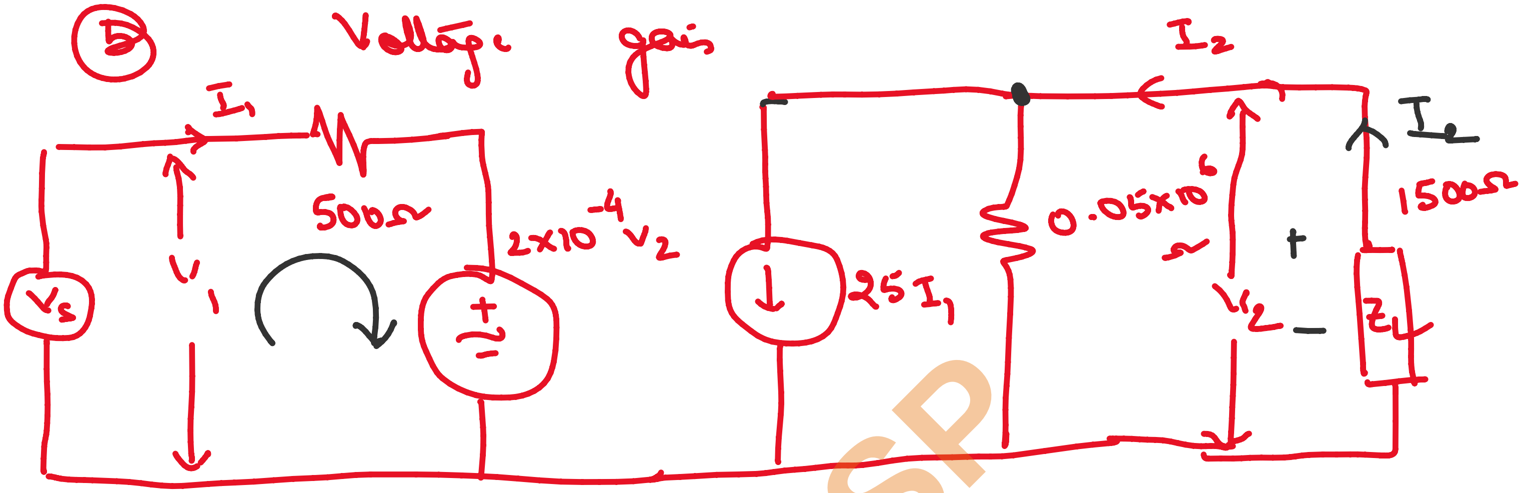
More problems on two port network parameters

16 January 2021 11:04

For a h-parameter eq. n/w shown,

(a) determine the current gain

(b) Voltage gain



$$I_2 = \frac{25 I_1 \times 0.05\times10^{6}}{1500 + 0.05\times10^{6}}$$

$$A_I = \frac{I_2}{I_1} = \frac{25\times0.05\times10^{6}}{(1500+0.05\times10^{6})} = 24.27 \quad — (1)$$

$$V_1 = 500 I_1 + 2\times10^{-4} V_2 \quad — (2) \qquad I_2 = \frac{-V_2}{1500} \quad — (3)$$

KCL at outer node $I_2 = 25 I_1 + \dfrac{V_2}{0.05\times10^{6}}$ — (4)

(3) in (4)

$$\frac{-V_2}{1500} = 25 I_1 + \frac{V_2}{0.05\times10^{6}} \quad — (5)$$

from eq (2) $\quad V_1 - 2\times10^{-4} V_2 = I_1$ — (6)

from eq ② $\frac{V_1 - 2\times10^{-4}V_2}{500} = I_1$ — ⑥

sub ⑥ in ⑧

$$-\frac{V_2}{1500} = 25\times\left(\frac{V_1 - 2\times10^{-4}V_2}{500}\right) + \frac{V_2}{0.05\times10^6}$$

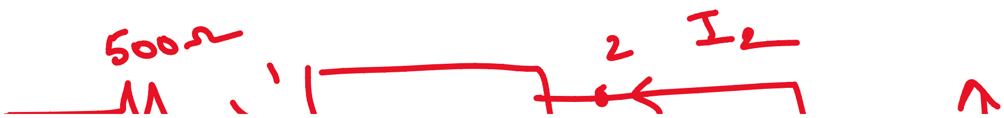
$$-\frac{V_2}{1500} = \frac{25V_1}{500} - \frac{25\times2\times10^{-4}}{500}V_2 + \frac{V_2}{0.05\times10^6}$$

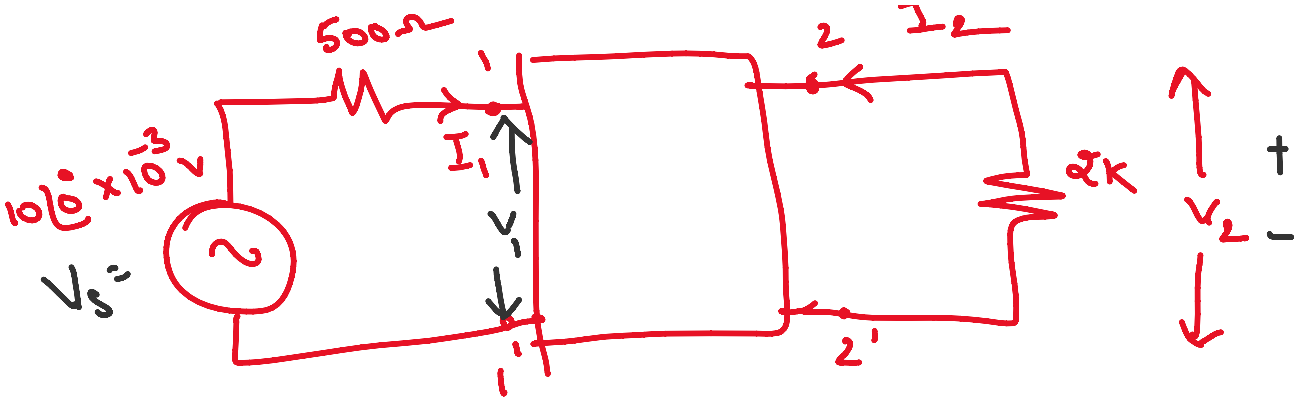
$$\frac{25V_1}{500} = V_2\left[\frac{-1}{1500} + \frac{25\times2\times10^{-4}}{500} - \frac{1}{0.05\times10^6}\right]$$

$$\frac{V_1}{20} = V_2\left[-6.77\times10^{-4}\right]$$

$$\frac{V_2}{V_1} = -73.89.$$

The hybrid parameters of a 2 port n/w shown in fig are $h_{11} = 1K$, $h_{12} = 0.03$, $h_{21} = 100$, $h_{22} = 50\mu℧$. Find V_2 and Z parameters of the n/w.





$$V_1 = h_{11} I_1 + h_{12} V_2 \quad -①$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -②$$

$$V_1 = 500 I_1 + 0.003 V_2 \quad -①$$

$$I_2 = 100 I_1 + 50\mu V_2 \quad -②$$

$$-I_2 2000 = V_2 \quad -③$$

Sub ③ in ②

$$I_2 = 100 I_1 - 50\mu \times 2000 I_2$$

$$I_2 \left[1 + 50\mu \times 2000\right] = 100 I_1$$

$$I_2 = \left(\frac{100}{1 + 50\mu \times 2000}\right) I_1 \Rightarrow \frac{I_2}{I_1} = \frac{100}{1.1} \quad -④$$

Sub ③ in ①

$$V_1 = 1k\, I_1 + 0.003 \left[-I_2 2000\right] \quad -⑤$$

$$V_s - 500 I_1 = V_1 \quad -⑥$$

eq ⑤ & ⑥ by eliminating V_1

$$V_s - 500 I_1 = 1K I_1 + 0.003 \left[- I_2 \, 2000 \right] \quad — ⑦$$

sub ④ in ⑦

$$V_s = 500 I_1 + 1K I_1 + 0.003 \left[- \frac{100}{1.1} I_1 \cdot 2000 \right]$$

$$10\angle 0^\circ \times 10^{-3} = 954.54 I_1$$

$$I_1 = \frac{10 \times 10^{-3}}{954.54} = \underline{10.5 \times 10^{-6} A}$$

$$V_1 = V_s - 500 I_1 = 10 \times 10^{-3} - 500 \times 10.5 \times 10^{-6}$$

$$= 4.75 \times 10^{-3} \, V$$

$$V_2 = \frac{V_1 - h_{11} I_1}{h_{12}} \quad (\text{first } h \text{ parameter eq } 5)$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

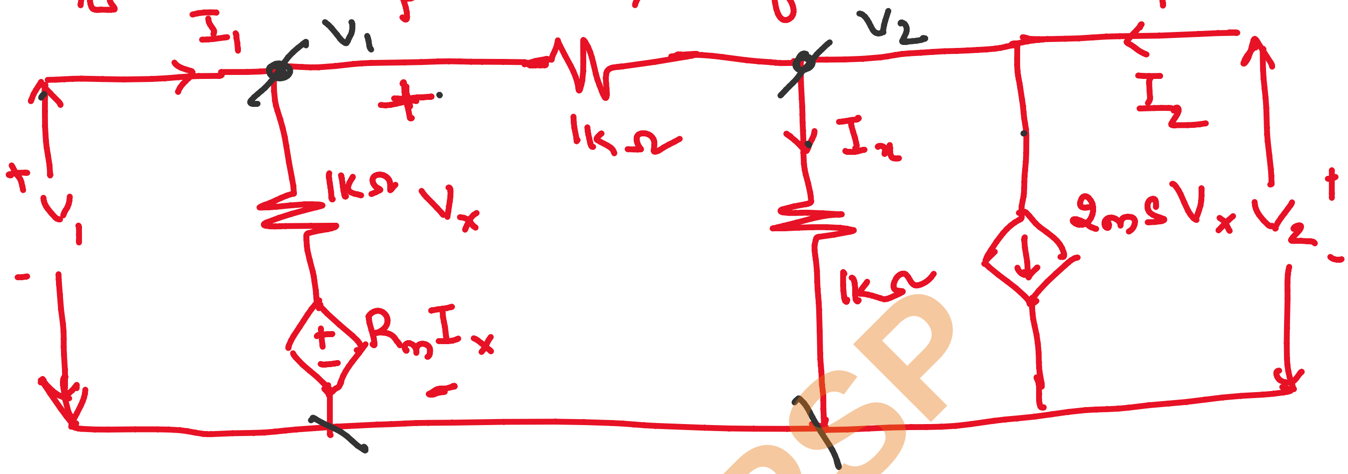
$$V_2 = \frac{4.75 \times 10^{-3} - 1K \times 10.5 \times 10^{-6}}{0.003} = -1.916 \, V.$$

$$Z_{11} = \frac{\Delta h}{h_{22}} \qquad Z_{21} = \frac{-h_{21}}{h_{22}}$$

$$Z_{12} = \frac{h_{12}}{h_{22}} \qquad Z_{22} = \frac{1}{h_{22}}$$

$$z_{12} = \frac{h_{12}}{h_{22}} \qquad z_{22} = \frac{1}{h_{22}}$$

(8) For the n/w shown below, find R_m if the n/w is reciprocal. Also find the h-parameters.



$$V_1 = V_x \quad -(1) \qquad I_x = \frac{V_2}{1k} \quad -(2)$$

Nodal at V_1

$$\frac{V_1 - R_m I_x}{1k} + \frac{V_1 - V_2}{1k} = I_1 \quad -(3)$$

Nodal at V_2

$$\frac{V_2 - V_1}{1k} + \frac{V_2}{1k} + 2mS\,V_x = I_2 \quad -(4)$$

sub (2) in (3) & (1) in (4)

$$\frac{V_1 - R_m\left(V_2 \times 10^{-3}\right)}{1k} + \frac{V_1 - V_2}{1k} = I_1$$

$$I_1 = \frac{2V_1}{1k} - \frac{V_2}{1k}\left[1 + R_m \times 10^{-3}\right]$$

$$I_1 = 2\times10^{-3} V_1 - 1\times10^{-3} V_2 \left[1 + R_m \times 10^{-3}\right] \quad -(5)$$

$$\frac{V_2 - V_1}{1k} + \frac{V_2}{1k} + 2m V_1 = I_2$$

$$I_2 = (2m - 1m) V_1 + V_2 (1m + 1m)$$

$$I_2 = 1m V_1 + 2m V_2 \quad - \quad (6)$$

$$I_1 = 2m V_1 - (R_m \times 10^{-6} + 1m) V_2$$

$$I_2 = 1m V_1 + 2m V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2m & -(R_m \times 10^{-6} + 1m) \\ 1m & 2m \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Std. Y-parameter format.

Condition for Reciprocity in Y parameters is

$$Y_{12} = Y_{21}$$

$$-R_m \times 10^{-6} - 1m = 1m$$

$$-R_m \times 10^{-6} = 2m$$

$$R_m = -\frac{2m}{10^{-6}} = \underline{-2K}$$

$$\begin{tikzcd} Y \arrow[r] \arrow[rr, bend right] & Z \arrow[r] & h \end{tikzcd}$$

$Z \to h$

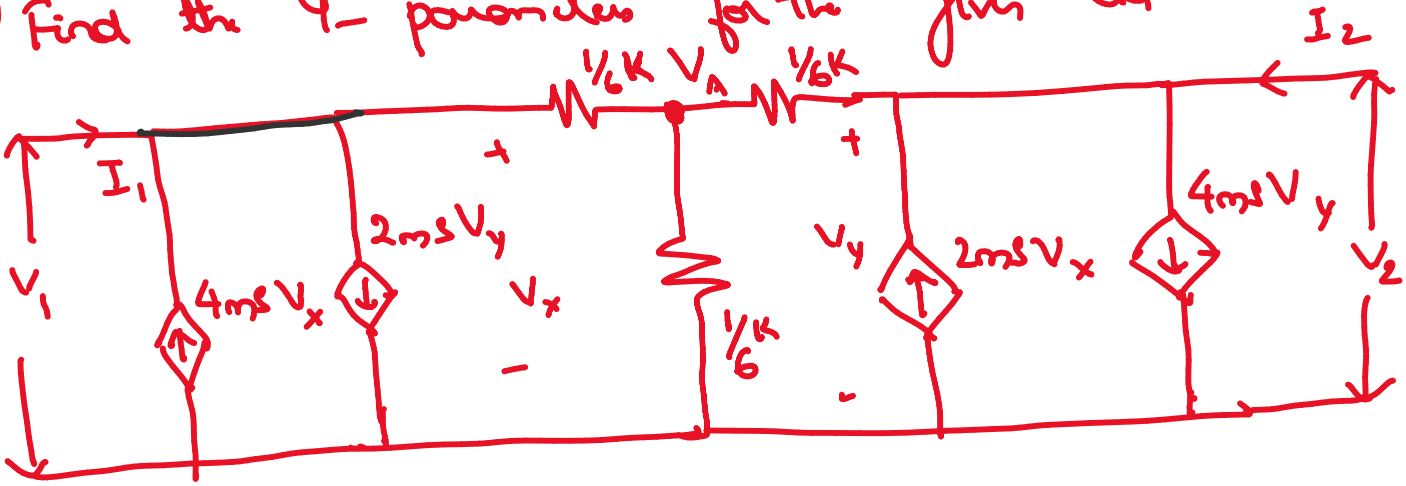
$h \to Z$

$Z \to \Delta$

$n \rightarrow z$

$z \rightarrow A$

9. Find the Y- parameters for the given ckt



$V_x = V_1, \quad V_y = V_2$

At V_1: $\quad I_1 = -4ms\,V_x + 2ms\,V_y + \dfrac{V_1 - V_A}{1/6K}$

$I_1 = -4ms\,V_1 + 2ms\,V_2 + (V_1 - V_A)6ms$

$I_1 = 2ms\,V_1 + 2ms\,V_2 - 6ms\,V_A \quad$ — (1)

At V_A: $\quad \dfrac{V_A - V_1}{1/6K} + \dfrac{V_A}{1/6K} + \dfrac{V_A - V_2}{1/6K} = 0$

$-6ms\,V_1 - 6ms\,V_2 + 18ms\,V_A = 0$

$V_A = \dfrac{V_1 + V_2}{3} \quad$ — (2)

At V_2: $\quad I_2 = +4ms\,V_y + 2ms\,V_x + \dfrac{V_2 - V_A}{1/6K}$

$I_2 = -4ms\,V_2 + 2ms\,V_1 + 6ms(V_2 - V_A)$

6K

$$I_2 = -4ms\,V_2 + 2ms\,V_1 + 6ms\,(V_2 - V_A)$$

$$I_2 = 2ms\,V_1 + 10ms\,V_2 + 6ms\,V_A \quad - \quad 3$$

$$V_A = \left(\frac{V_1 + V_2}{3}\right)$$

sub 2 in 1

$$I_1 = 2ms\,V_1 + 2ms\,V_2 - 6ms\,V_A$$

$$= 2ms\,V_1 + 2ms\,V_2 - 6ms\left[\frac{V_1 + V_2}{3}\right]$$

$$I_1 = 0V_1 + 0V_2 \quad - \quad 4$$

$$I_2 = -2ms\,V_1 + 10ms\,V_2 - 6ms\left[\frac{V_1 + V_2}{3}\right]$$

$$= -2ms\,V_1 + 10ms\,V_2 - 2ms\,V_1 - 2ms\,V_2$$

$$I_2 = -4ms\,V_1 + 8ms\,V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -4ms & +8ms \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_2 - \left(\frac{V_2 - V_A}{1/6K}\right) + 2ms\,V_x - 4ms\,V_y = 0$$

$$\frac{1}{2}\left(\frac{v_2 - v_1}{\frac{1}{6}k}\right) + 2msV_x - 4msV_y = 0$$

Network functions

09 January 2021 10:38

Driving pt Impedance

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

$V_1(s) \rightarrow$ Roots ~~of~~ of $V_1(s)$ are zeros

$I_1(s) \rightarrow$ Roots of $I_1(s)$ are poles.

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

Short ckt

If $Z_{11}(s) = 0$, $V = 0$ (The zeros of $V(s)$ polynomial lead of zeros of $Z(s)$)

$Z_{11}(s) = \infty$, $I = 0$ (The zeros of $I(s)$ polynomial lead of poles of $Z(s)$)

Open ckt

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$

's' - domain

Time domain

's' - domain

N/w parameter

$$V_2(s) = \boxed{G_{21}(s)\, V_1(s)}$$

I/p parameter

$$V_2(s) = \boxed{\quad} + \boxed{\quad}$$

Natural ✓ Forced ✓

$$\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}$$

$$I_2(s) = \alpha_{21}(s)\, I_1(s)$$

N/w parameter I/p parameter

$R + sL$

$R + \frac{1}{sC}$

$sL + \frac{1}{sL}$

$s(s^2+1)$ $s = \pm j1$

$s(s \pm j1)$

R	sL	$\frac{1}{sC}$

RL
RC
LC

$$\frac{\cancel{(s^2+2s+5)}}{}$$

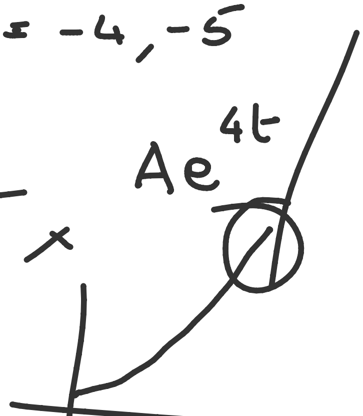
$$\frac{(s+2)(s+3)}{(s+4)(s+5)}$$

Zeros $s = -2, -3$

poles $s = -4, -5$

$\frac{A}{s-4}$ × Ae^{4t}

'∞'



unstable

$\frac{A}{s+4} + \frac{B}{s+5}$

$P = -4$

$Ae^{-4t} \rightarrow$ Stable

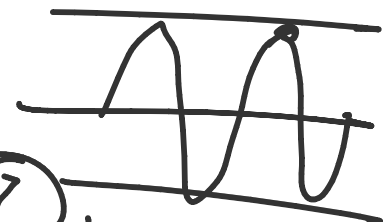
e^{pt} At $t = \infty$, $y(t) = 0$

Pi(-ve

unstable   Pi (-ve) $=0$

$$\frac{A}{(s^2+a^2)^2}$$

$t \sin \omega t$ ✗ $t \cos \omega t$



$\sin \omega t$ → marginally

$$\frac{A}{(s^2+a^2)(s^2+b^2)}$$

∞

Z_{12} : Resistor $= R\,\Omega$ (Zero order)

$(j\omega L)\ \Omega = (\Omega) =$ Inductance : Ls (1)

$\frac{1}{j\omega C}\ (\Omega) =$ Capacitance $= \frac{1}{Cs}$ (1)

$\underbrace{R,\ Ls,\ \frac{1}{Cs}}$ → Valid Resistance eq. in S domain

$S = \sigma + j\omega$

$s \propto \omega$ $s^2 \propto \omega^2$ $s^{\infty} \propto \omega^{\infty}$

2 4 s^n & ω^n

2^n. s

s^n

$$Z(s) = \lim_{s\to\infty} \left[\frac{a_0 s^n + \dots}{b_0 s^m + \dots}\right] = \lim_{s\to\infty} \frac{a_0 s^n}{b_0 s^m}$$

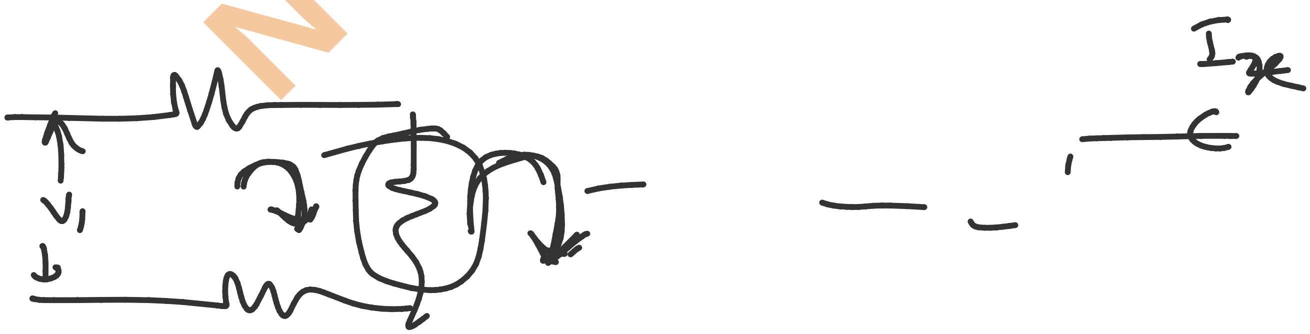
$$= \lim_{s\to\infty} \frac{a_0}{b_0} s^{n-m} = R, \quad = Ls, \quad = \frac{1}{Cs}$$

$n-m=0$

$n-m=1$

$n-m=-1$

Highest order must diff by 0 or 1



$\frac{I_x}{V_1}$

$\underbrace{R_3(I_1 - I_2)}$

Introduction

03 October 2020 10:28

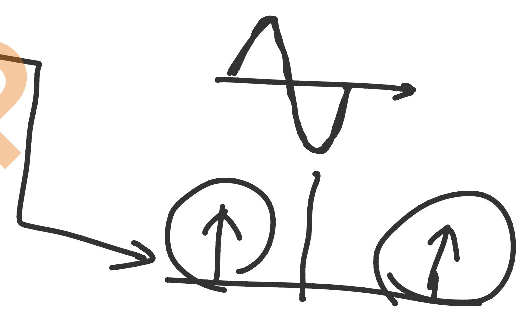
1) Frequency Domain study ?

F.T , F.S etc → Freq. domain.

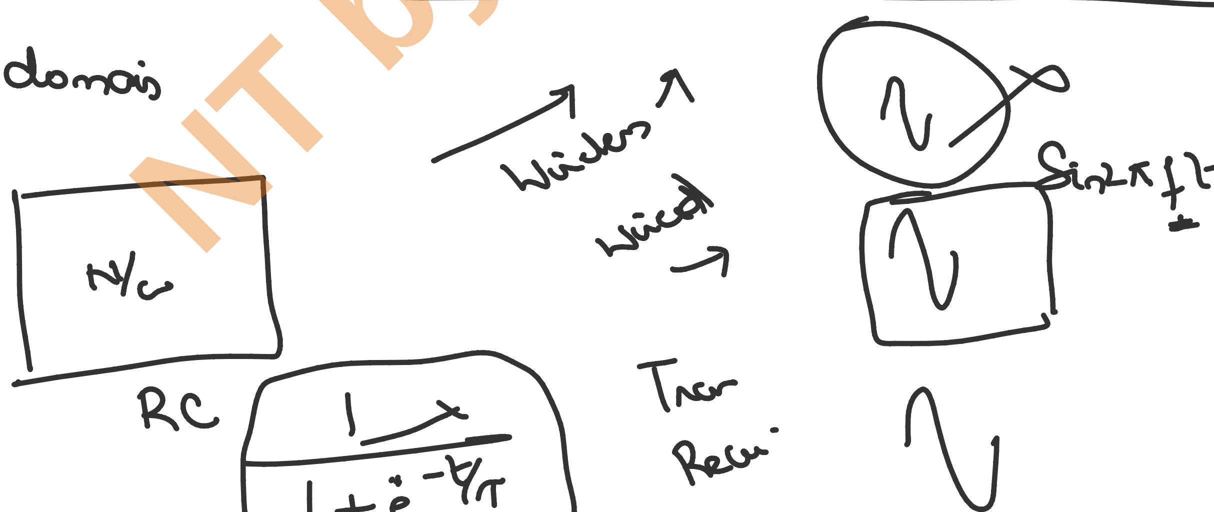
Time domain — To visual is easy. Signal change w.r.t. time

For analysis — Freq. domain ?

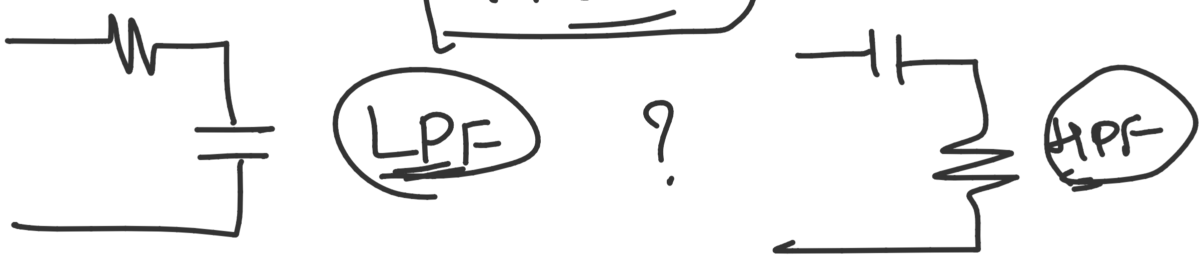
FM — MHz



Freq. domain



$\frac{1}{1+e^{-t/\tau}}$



Freq. Domain — ① F.T — Fourier Transform

freq. Components?
How will they be affected when passed thro' a particular system?
→ HP? B?
→ LP?

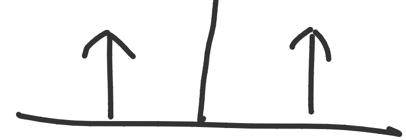
---

① FT

$$F(\omega) = \frac{1}{T}\int_{t_1}^{t_2} e^{-j\omega t} f(t)\,dt$$

$e^{-j\omega t} = \cos\omega t - j\sin\omega t$

$f(t)$ — Signal in time domain



$$\boxed{\int_0^\infty f(t)\,dt = \cancel{\infty}\ \text{Finite Value}}$$

$$\int_0^\infty u(t)\,dt = t\Big|_0^\infty = \infty$$

$$\int \alpha t\,dt = \frac{\alpha t^2}{2} = \infty$$

$$\int e^{\alpha t} = \frac{e^{\alpha t}}{\alpha} = \infty$$

② $\ast$ $\int_{0^-}^{\infty}$

Time domain

$v_c(0^-)$ $i_L(0^-)$

$\times$ $\times$

$$\int_{-\infty}^{\infty} \quad \Big| \quad \int_{0}^{\infty}$$

Laplace Transform :-

$$L[f(t)] = \int_{-\infty}^{\infty} \left[e^{-j\omega t} \boxed{f(t)}\right] \boxed{e^{-\sigma t}} \, dt \; *$$

$e^{-\infty} = \underline{\underline{0}}$

$0 \times \frac{1}{0}$

① $$L[f(t)] = \int_{-\infty}^{\infty} e^{-(\sigma + j\omega)t} f(t)$$

$$= \int_{-\infty}^{\infty} e^{-st} f(t) \qquad s = \sigma + j\omega$$

Causal System

② $$L[f(t) = \int_{\boxed{0^-}}^{\infty} e^{-st} f(t)$$

[Laplace transform covers the Initial Conditions]

③ $$\underbrace{\int_{0^-}^{0} e^{-st} f(t)}_{\text{Initial condition}} + \int_{0}^{\infty} e^{-st} f(t)$$

* Absolute Integratable *

Initial Value Theorem :-

$$* \quad \overbrace{f(0^-) = \lim s F(s)}^{} \quad *$$

Initial Value at $\underline{t_m = 0}$

* $f(0^-) = \lim_{s \to \infty} sF(s)$ * $t_n = 0$

Final Value Theorem

* $f(\infty) = \lim_{s \to 0} sF(s)$ *

Steady state Response is TD

Operational Representation of Circuit Elements

03 October 2020 10:51

Resistor

$$v_R(t) = i_R(t) \times R$$

G - Conductance

$$i_R(t) = \frac{v_R(t)}{R} = G v_R(t)$$

Laplace Domain

$$V_R(S) = I_R(s).R \quad \text{or} \quad I_R(s) = G V_R(s)$$

$$R = \frac{V_R(s)}{I_R(s)} \quad \text{or} \quad G = \frac{I_R(s)}{V_R(s)}$$

Inductance

$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$\Rightarrow \text{or} \quad i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt = \frac{1}{L}\int_{0^-}^{t} v_L(t)\,dt$$

$$* \quad I_L(s) = \frac{V_L(s)}{Ls} + \left[\frac{i_L(0^-)}{s}\right] \rightarrow \text{Initial value} \quad — (1)$$

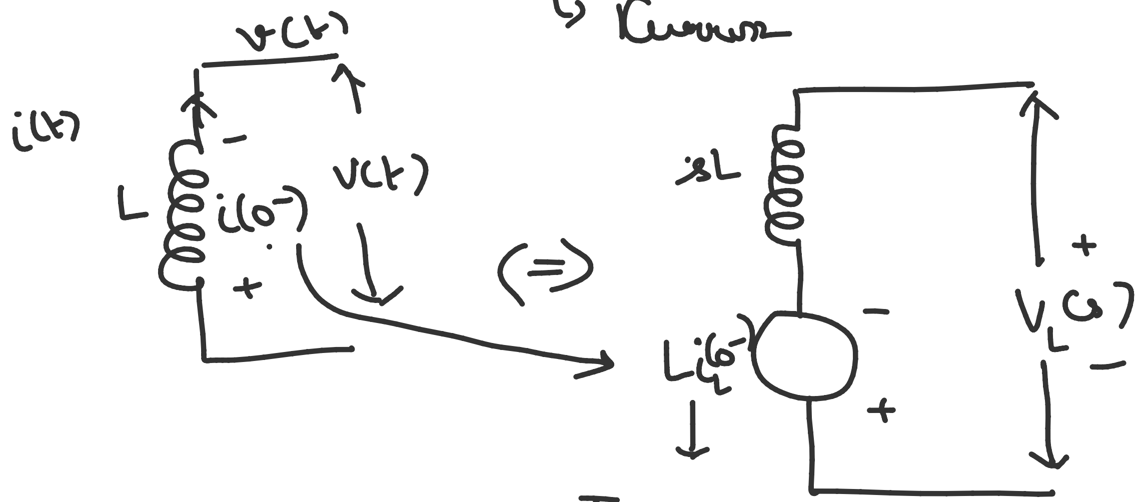
$$\Rightarrow V_L(s) = sL\,I(s) - L i_L(0^-)$$

$$\Rightarrow * \; V(s) = L[s I_L(s) - i_L(0^-)] \quad — (2)$$

$V_L(s)$ & $I(s)$ — The i/p that is used.
↳ Current

$v(t)$

↳ Current



Transformed Initial value

Capacitor

$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt \quad \text{or} \quad i(t) = C\frac{dv_c(t)}{dt}$$

$$\downarrow$$

$$V_c(s) = \frac{1}{C}\left[\frac{I(s)}{s} + \frac{q(0^-)}{s}\right]$$

$$\boxed{\begin{aligned} V &= \frac{Q}{C} \\ Q &= VC \end{aligned}}$$

$$i = \frac{dq}{dt}$$

$$\int i = q$$

$$= \frac{I(s)}{Cs} + \left[\frac{q(0^-)}{C}\right]s = \frac{I(s)}{Cs} + \frac{V_0}{s}$$

$$V_c(s) = \frac{I(s)}{Cs} + \frac{V_0}{s} \quad \text{— (1)}$$

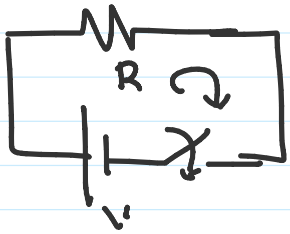
$$\frac{I(s)}{Cs} = V_c(s) - \frac{V_0}{s}$$

$$I(s) = CsV(s) - CV_0 \quad \text{— (2)}$$

Transformed Initial Voltage

DC excitation

03 October 2020 11:06



$V = iR$

$V(s) = I(s)R$

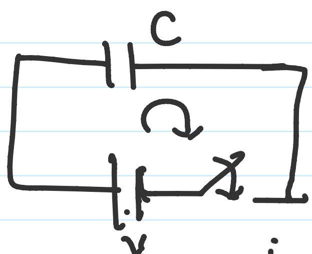
$I(s) = \frac{V(s)}{R} = \frac{V}{sR}$

$V(s) = L[V]$

$= V/s$

sL

$\boxed{\frac{dv}{dt} = \frac{1}{s} \cdot L(v)}$

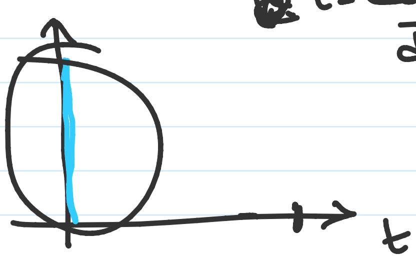


$i = C\frac{dv}{dt}$

$\Downarrow$

$= I(s) \frac{1}{Cs} \cdot V(s) = \frac{V/s}{Cs}$

$i = C\frac{dv}{dt}$

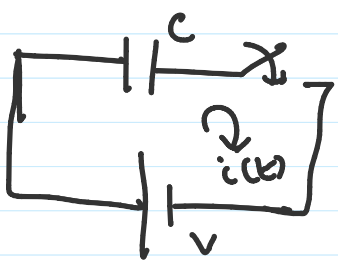


$\Rightarrow I(s) = VC$

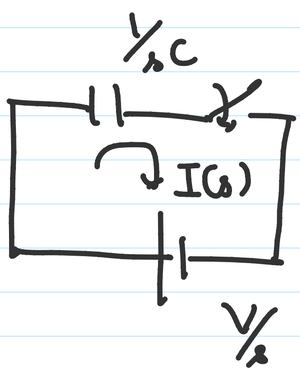
$i(t) = VC(\delta(t))$

$i(0) = i(t) = \lim_{s\to\infty} sI(s) = \lim_{s\to\infty} sVC = \infty$

$i(\infty) = \lim_{s\to 0} sI(s) = \lim_{s\to 0} sVC = 0$



$\Rightarrow$



$v(t) = V$

$V(s) = V/s$

$C = \frac{1}{j\omega C}$

$= \frac{1}{sC}$

$i = C\frac{dv}{dt}$

$= C \cdot s \cdot L[v(t)]$

$$= C \cdot s \cdot L[v(t)] \qquad // \qquad = \frac{1}{sC}$$

$$I(s) = Cs \cdot V/s = CV$$

$$I(s) = \frac{V/s}{1/sC} = \underline{VC}$$

$$\Rightarrow \quad i(t) = VC\,\delta(t)$$

$$\boxed{L^{-1}[1] = \delta(t)}$$

Initial Value Theorem

$$i(0^+) = \lim_{s\to\infty} s\,I(s) = \lim_{s\to\infty} sVC = \infty$$

$$\boxed{'\infty'}$$

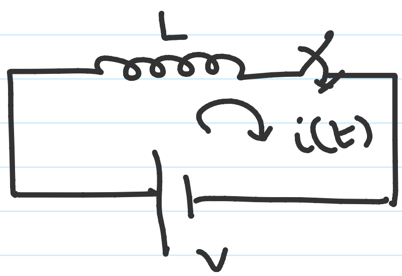
If no voltage is present in Cap at $t=0^-$ at $t=0^+$ $v_c(0^+) = 0$ 'SC'

Final Value Theorem

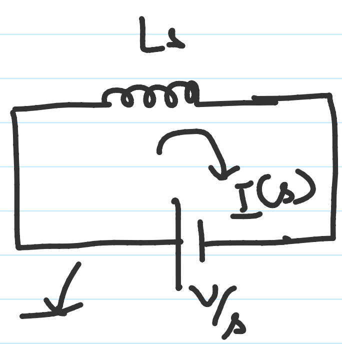
$$\left[\, i = \frac{dv}{dt} = 0 \quad \text{when} \quad v_c = V \,\right] \quad = \underline{0}$$

$$i(\infty) = \lim_{s\to 0} s\,I(s) = \lim_{s\to 0} sVC = 0$$

Inductance:-



$\Rightarrow$



$$I(s) = \frac{V/s}{Ls} = \frac{V}{Ls^2} \quad - (1)$$

$$i(t) = \frac{1}{L}\int v\,dt \Rightarrow I(s) = \mathcal{L}\left[\frac{1}{L}\int v\,dt\right]$$

$$= \frac{1}{L}\mathcal{L}\left[\int v\,dt\right] = \frac{1}{L}\cdot\frac{1}{s}\mathcal{L}\left(v(t)\right)$$

$$\boxed{\mathcal{L}\int f(t) = \frac{1}{s}\mathcal{L}[f(s)]}$$

✓ $I(s) = \frac{1}{Ls}\cdot V/s = \frac{V}{Ls^2}$ — (2)

Time domain Rep. of current $i(t) = \mathcal{L}^{-1}\left[\frac{V}{Ls^2}\right] = V/L\, t$ — (3).

$\mathcal{L}[t] = 1/s^2$

✓ $i(0^-)$ through L is 0. $i(0^+)$ is also 0.

Initial Value Theorem

$$i(0^+) = \lim_{s\to\infty} s\,I(s) = \lim_{s\to\infty} \cancel{s}\cdot\frac{V}{Ls^{\cancel{2}}} = 0.$$

Final value theorem

$$i(\infty) = \lim_{s\to 0} s\,I(s) = \lim_{s\to 0} s\cdot\frac{V}{Ls^2} = \frac{V}{Ls} = \underline{\underline{\infty}}$$

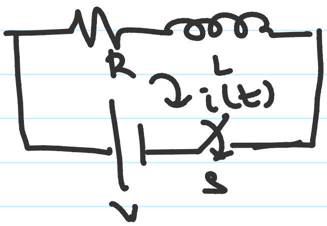
Because L acts as SC at $t = \infty$

$$\left[v = L\frac{di}{dt} = 0\right].$$

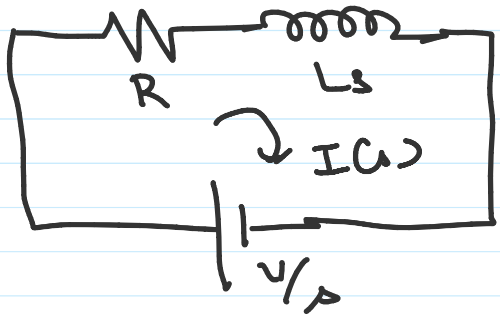
DC Excitation for RL and RC circuits

08 October 2020 10:22

RL Circuit :-



$\Rightarrow$



$$I(s) = \frac{V/s}{R+Ls} = \frac{V}{s(R+Ls)} = \frac{V}{sL\left[s+\frac{R}{L}\right]}$$

$$I(s) = \frac{V/L}{s\left(s+\frac{R}{L}\right)}$$

$$I(s) = \frac{A}{s} + \frac{B}{\left(s+\frac{R}{L}\right)} \quad \text{— ①}$$

$$A = \lim_{s\to 0} \frac{\cancel{s}\cdot V/L}{\cancel{s}\left(s+\frac{R}{L}\right)} = V/R \quad \text{— ②}$$

$$B = \lim_{s\to -R/L} \cdot \frac{\cancel{\left(s+\frac{R}{L}\right)}\, V/L}{s\,\cancel{\left(s+\frac{R}{L}\right)}} = \frac{V/L}{-\frac{R}{L}} = -\frac{V}{R} \quad \text{— ③}$$

Sub ② & ③ in ①

$$L^{-1}\left[\frac{1}{s}\right] = 1$$

$$L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}$$

$$I(s) = \frac{V}{Rs} - \frac{V}{R\left(s+\frac{R}{L}\right)}$$

$$i(t) = \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t}$$

$$i(t) = V/R - \frac{V}{R} e^{-R/L\, t}$$

$$i(t) = \frac{V}{R}\left[1 - e^{-\frac{R}{L}t}\right] = \frac{V}{R}\left[1 - e^{-t/\tau}\right]$$

$$\tau = L/R$$

From Initial Value Theorem

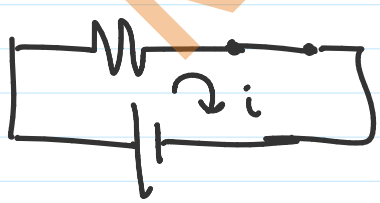
$$i(0+) = \lim_{s\to\infty} \cancel{s} \cdot \frac{V/L}{\cancel{s}(s + R/L)} = \frac{V/L}{\infty} = 0.$$

[Because of inductor $i(0^-) = 0$, $i(0+) = 0$.]

From final value theorem

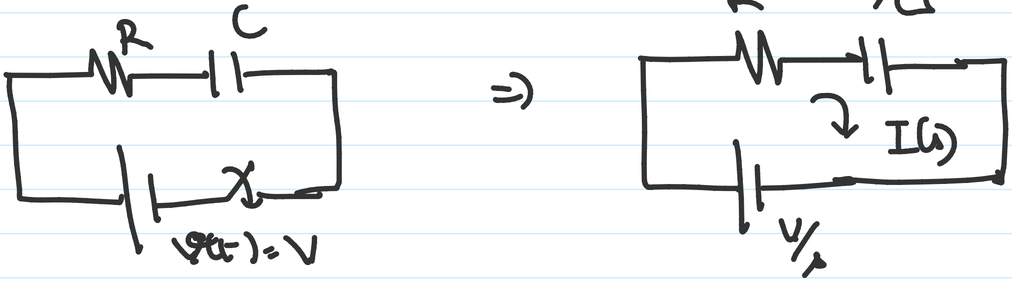
$$i(\infty) = \lim_{s\to 0} \cancel{s} \cdot \frac{V/L}{\cancel{s}(s + R/L)} = V/R$$

[At $t = \infty$, L acts as a SC, so the voltage V appears across R $\Rightarrow$ $i(\infty) = V/R$]



$i = V/R.$

R C Circuit :-



$$I(s) = \frac{V/s}{\quad} = \frac{V/s}{\quad}$$

$$I(s) = \frac{V/s}{R + 1/Cs} = \frac{V/s}{\frac{RCs+1}{Cs}} =$$

$$I(s) = \frac{V}{s} \cdot \frac{Cs}{RCs+1} = \frac{VC}{RCs+1} = \frac{VC}{RC\left[s + \frac{1}{RC}\right]}$$

$$I(s) = \frac{V/R}{\left(s + \frac{1}{RC}\right)} \qquad L^{-1}\left[\frac{A}{s+a}\right] = Ae^{-at}$$

$$i(t) = \frac{V}{R} e^{-\frac{1}{RC} t} = \frac{V}{R} e^{-t/RC} = \underline{\frac{V}{R} e^{-t/\tau}}$$

$$I(s) = \frac{V/R}{s + 1/RC}$$

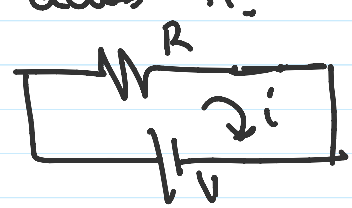
$$i(0^+) = \lim_{s \to \infty} sI(s) = \lim_{s \to \infty} \frac{s \cdot V/R}{s + \frac{1}{RC}} = \lim_{s \to \infty} \frac{\cancel{s}\left[V/R\right]}{\cancel{s}\left[1 + \frac{1}{RCs}\right]}$$

$$= \frac{V/R}{1+0} = V/R.$$

At $t=0^-$, $v_c(0^-)=0$ so $t=0^+$ also $v_c(0^+)=0$.

so the whole voltage appears across R.

$\Rightarrow$ $i = V/R$.



$$i(\infty) = \lim sI(s) = \lim s \cdot \frac{V/R}{\ldots} = 0$$

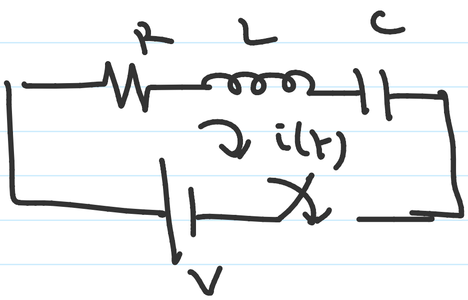
$$i(\infty) = \lim_{s \to 0} s\, I(s) = \lim_{s \to 0} s \cdot \frac{V/R}{s + 1/RC} = 0$$

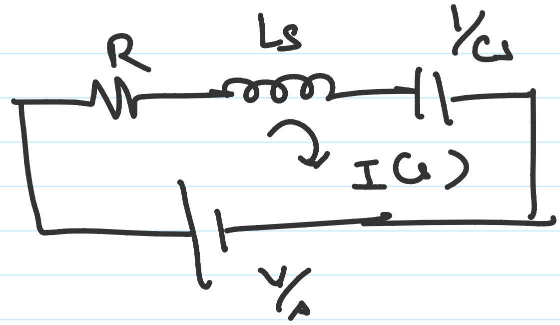
At $t = \infty$, Capacitor behaves like OC. not allowing any current

$i = C\frac{dv}{dt} \Rightarrow i = 0.$

DC excitation for RLC Circuit

08 October 2020 11:02





$$I(s) = \frac{\frac{V}{s}}{R + Ls + \frac{1}{Cs}} = \frac{V/s}{\frac{RCs + CLs^2 + 1}{Cs}}$$

$$= \frac{V}{s} \cdot \frac{Cs}{1 + LCs^2 + RCs} = \frac{VC}{LC\left[s^2 + \frac{RC}{LC}s + \frac{1}{LC}\right]}$$

$$I(s) = \frac{V/L}{\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

$$s = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$I(s) = \frac{V/L}{\left[s + \frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right]\left[s + \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right]}$$

If $\frac{R}{2L} = a$ & $b = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$$\Rightarrow \quad I(s) = \frac{V/L}{(s+a-b)(s+a+b)}$$

$$I(s) = \frac{V/2bL}{(s+a-b)} - \frac{V/2bL}{(s+a+b)}$$

$$i(t) = \frac{V}{2bL}\left[e^{-(a-b)t} - e^{-(a+b)t}\right] \quad —— ①$$

If $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$, b is real

If $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$, b is zero

If $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$, b is imaginary.

Case 1:- b is real, positive

$$i(t) = \frac{V}{2bL}\left[e^{-(a-b)t} - e^{-(a+b)t}\right]$$

Case 2:- b is zero

$$\Rightarrow i(t) = \frac{V}{2bL}\left[e^{-at} - e^{-at}\right]$$ an indeterminant

Diff i(t) wrt t.

$$i(t) = \frac{V}{L} t e^{-at}.$$

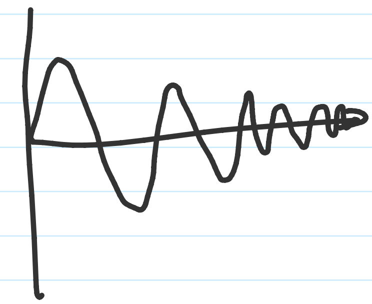
Case 3:- If b is imaginary

$$i(t) = \frac{V}{\;}\left(e^{-at}e^{jbt} \quad e^{-at}e^{-jbt}\right)$$

$$i(t) = \frac{V}{2bL}\left(e^{-at}e^{jbt} - e^{-at}e^{-jbt}\right)$$

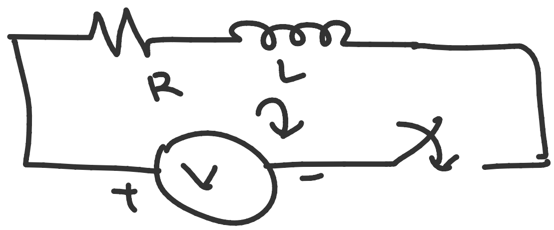
$$= \frac{V}{2bL}e^{-at}\left(e^{jbt} - e^{-jbt}\right)$$

$$= \frac{V}{2bL}e^{-at} \cdot 2 \cdot \sin bt$$



Sin excitation for RL circuits

08 October 2020 11:13



$$v(t) = V_m \sin(\omega t + \phi)$$

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V_m}{R + sL}\left[L[\sin \omega t] \cos\phi + L[\cos \omega t] \sin\phi \right]$$

$$I(s) = \frac{V_m}{R + sL}\left[\frac{\omega}{s^2 + \omega^2} \cdot \cos\phi + \frac{s}{s^2 + \omega^2} \sin\phi \right]$$

Let $\frac{R}{L} = a$.

$$I(s) = \frac{V_m}{L\left(s + \frac{R}{L}\right)}\left[\frac{\omega}{(s^2 + \omega^2)} \cos\phi + \frac{s}{s^2 + \omega^2} \sin\phi \right]$$

$$= \frac{V_m / L}{(s + a)}\left[\frac{\omega \cos\phi}{(s^2 + \omega^2)} + \frac{s \sin\phi}{(s^2 + \omega^2)} \right]$$

$$I(s) = \frac{V_m}{L}\left[\frac{\omega \cos\phi}{(s + a)(s^2 + \omega^2)} + \frac{s \sin\phi}{(s + a)(s^2 + \omega^2)} \right]$$

$$\frac{1}{(s + a)(s^2 + \omega^2)} = \frac{1}{(a^2 + \omega^2)}\left[\frac{1}{(s + a)} + \frac{a}{s^2 + \omega^2} - \frac{s}{s^2 + \omega^2} \right]$$

$$\frac{\quad}{(s+a)(s^2+\omega^2)} = \frac{\quad}{(a^2+\omega^2)}\left[\frac{\quad}{(s+a)} \quad \frac{\quad}{s^2+\omega^2} \quad \frac{\quad}{s^2+\omega^2}\right]$$

$$\frac{s}{(s+a)(s^2+\omega^2)} = \frac{1}{(a^2+\omega^2)}\left[\frac{as}{s^2+\omega^2} + \frac{\omega^2}{s^2+\omega^2} - \frac{a}{s+a}\right]$$

$$i(t) = \frac{V_m}{(a^2+\omega^2)\cdot L}\left\{\omega\cos\phi\left(e^{-at} + \frac{a}{\omega}\sin\omega t - \cos\omega t\right) + \sin\phi\left[a\cos\omega t + \omega\sin\omega t - a e^{-at}\right]\right\}$$

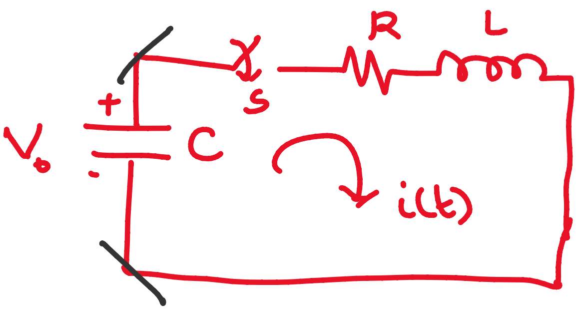
Problems on Laplace

15 December 2020 13:22

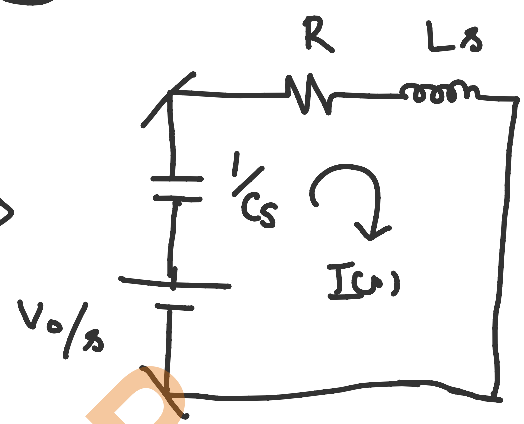
$$V(s) = sL\,I(s) - L\,i(0^-) \quad \text{— (1)}$$

$$V(s) = \frac{I(s)}{sC} + \frac{v_c(0^-)}{s} \quad \text{— (2)}$$

(1)



$\Leftrightarrow$



$$R\,I(s) + Ls\,I(s) + \frac{1}{Cs}I(s) = \frac{V_0}{s}$$

$$\left(R + Ls + \frac{1}{Cs}\right) I(s) = \frac{V_0}{s}$$

$$I(s) = \frac{V_0/s}{R + Ls + 1/Cs} = \frac{V_0}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

$$= \frac{V_0}{L} \cdot \frac{1}{(s+a)^2 + b^2} = \frac{V_0}{L} \cdot \frac{1}{(s^2 + 2as + a^2 + b^2)}$$

$$a = \frac{R}{2L}, \quad b = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

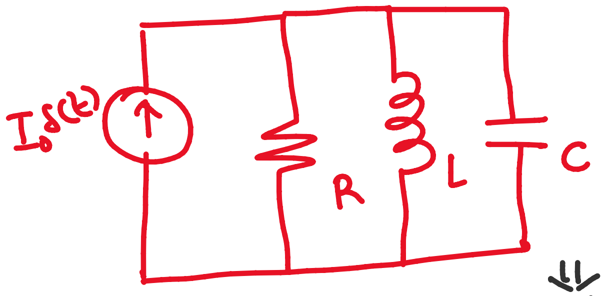
$$I(s) = \frac{V_0 \cdot b}{bL \left((s+a)^2 + b^2\right)} = \frac{V_0}{bL} \cdot \frac{b}{(s+a)^2 + b^2}$$

$$i(t) = \frac{V_0}{bL} \cdot e^{-at} \sin bt$$

(2)

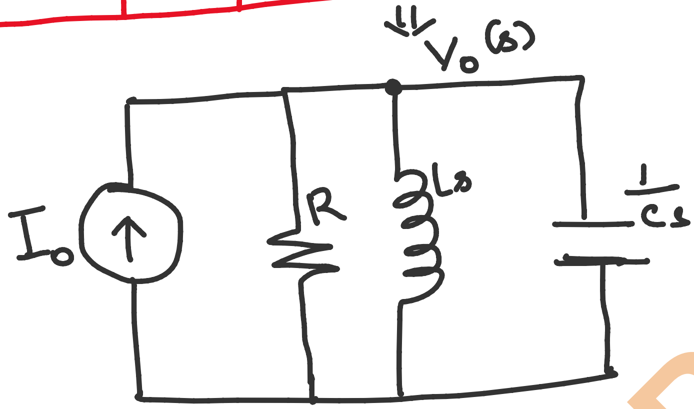
All initial conditions

②



$\Rightarrow$

All initial conditions taken to be zero the ckt can be drawn in Laplace domain as follows



$$\frac{V_0(s)}{R} + \frac{V_0(s)}{Ls} + \frac{V_0(s)}{1/Cs} = I_0$$

$$\Rightarrow \quad V_0(s)\left[\frac{1}{R} + \frac{1}{Ls} + Cs\right] = I_0$$

$$\Rightarrow \quad V_0(s)\left[\frac{Ls + R + RLCs^2}{RLs}\right] = I_0$$

$$\Rightarrow V_0(s) = \frac{I_0 . RLs}{RLCs^2 + Ls + R}$$

Divide the Nr & Dr with RLC

$$V_0(s) = \frac{\frac{I_0}{C}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

$$= \frac{I_0}{C}s$$

$$= \frac{\frac{I_0}{C}\, s}{s^2 + 2\cdot s\cdot\frac{1}{2RC} + \left(\frac{1}{2RC}\right)^2 + \frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

$$= \frac{I_0/C\; s}{\left(s+\frac{1}{2RC}\right)^2 + \left(\sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}\right)^2}$$

$$V_0(s) = \frac{I_0/C\; s}{(s+a)^2 + b^2}$$ where $a = \frac{1}{2RC}$

$$b = \sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}$$

$$V_0(s) = \frac{I_0/C\; s}{(s+a)^2+b^2} = \frac{I_0}{C}\cdot\frac{s}{(s+a)^2+b^2}$$

$$= \frac{I_0}{C}\cdot\left[\frac{(s+a)-a}{(s+a)^2+b^2}\right]$$

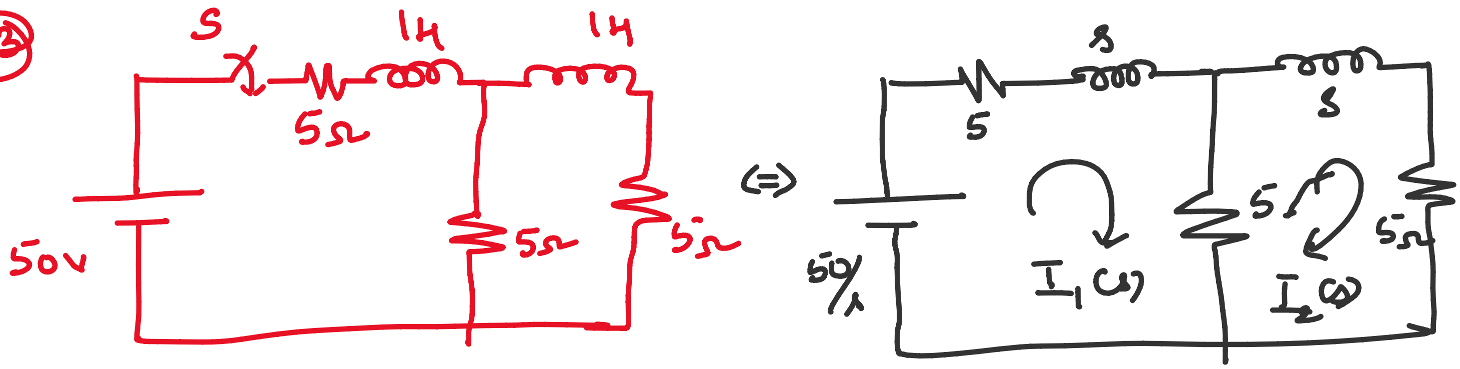
$$= \frac{I_0}{C}\left[\frac{s+a}{(s+a)^2+b^2}\right] - \frac{I_0}{C}\frac{a}{b}\cdot\frac{b}{(s+a)^2+b^2}$$

$\to$ Cos bt $\qquad$ $\to$ Sin bt

$$v_0(t) = \frac{I_0}{C}\left[e^{-at}\,\mathrm{Cos}\,bt\right] - \frac{I_0}{C}\cdot\frac{a}{b}\cdot e^{-at}\,\mathrm{Sin}\,bt$$

$$v_o(t) = \frac{I_o}{c}\left[e^{-at}\cos bt - \frac{a}{b}e^{-at}\sin bt\right]$$

③



$$\frac{50}{s} = 5I_1(s) + sI_1(s) + 5\left(I_1(s) - I_2(s)\right) \quad -①$$

$$5\left(I_2(s) - I_1(s)\right) + sI_2(s) + 5I_2(s) = 0 \quad -②$$

$$(10+s)\,I_1(s) - 5I_2(s) = 50/s$$

$$-5I_1(s) + (10+s)\,I_2(s) = 0$$

$$\begin{bmatrix} 10+s & -5 \\ -5 & 10+s \end{bmatrix}\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} 50/s \\ 0 \end{bmatrix}$$

$$I_1(s) = \frac{\begin{vmatrix} 50/s & -5 \\ 0 & 10+s \end{vmatrix}}{\begin{vmatrix} 10+s & -5 \\ -5 & 10+s \end{vmatrix}} = \frac{\frac{50}{s}(10+s)}{(10+s)^2 - 25}$$

$$I_1(s) = \frac{50(s+10)}{s\left[s^2+20s+100-25\right]} = \frac{50(s+10)}{s\left[s^2+20s+75\right]}$$

$$= \frac{50(s+10)}{s\left[s^2+15s+5s+75\right]} = \frac{50(s+10)}{s\left[s(s+15)+5(s+15)\right]}$$

$$I_1(s) = \frac{50(s+10)}{s(s+5)(s+15)} = \frac{A}{s} + \frac{B}{(s+5)} + \frac{C}{(s+15)}$$

$$A = \lim_{s\to 0} \cancel{s}\cdot\frac{50(s+10)}{\cancel{s}(s+5)(s+15)} = \frac{500}{5\times 15} = \frac{20}{3}$$

$$B = \lim_{s\to -5} \frac{\cancel{(s+5)}\,50(s+10)}{s\cancel{(s+5)}(s+15)} = \frac{50(-5+10)}{(-5)(-5+15)} = \frac{-\cancel{50}^{5}\times\cancel{5}}{\cancel{5}\times\cancel{10}}$$

$$= -5$$

$$C = \lim_{s\to -15} \frac{\cancel{(s+15)}\,50(s+10)}{s(s+5)\cancel{(s+15)}} = \frac{50(-15+10)}{-15\times(-15+5)} = \frac{50\times -5}{-15\times(-10)}$$

$$= -\frac{5}{3}$$

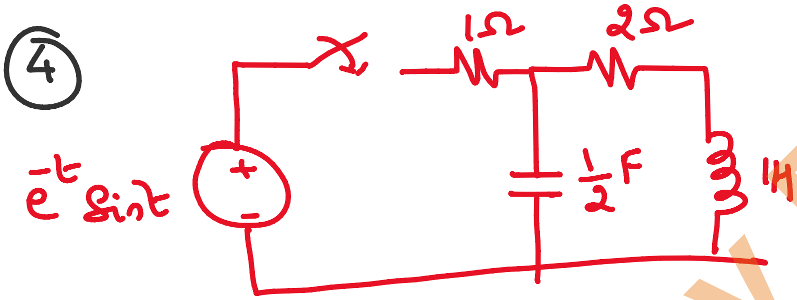
$$I_1(s) = \frac{20}{3s} - \frac{5}{s+5} - \frac{5}{3(s+15)}$$

$$i_1(t) = \frac{20}{3}u(t) - 5e^{-5t} - \frac{5}{3}e^{-15t}$$

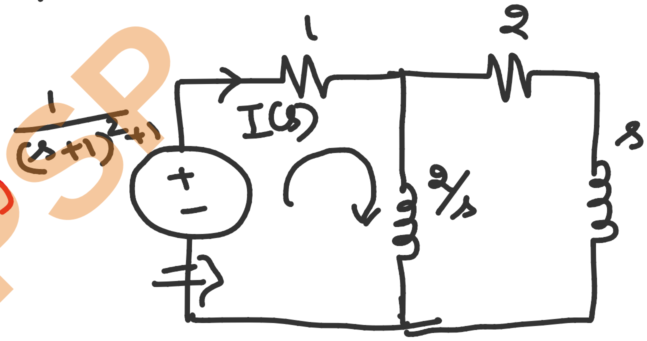
$$I_2(s) = \frac{10}{3s} - \frac{5}{s+5} + \frac{1.67}{s+15}$$

$$i_2(t) = \frac{10}{3}u(t) - 5e^{-5t} + 1.67e^{-15t}$$

4



$\Rightarrow$



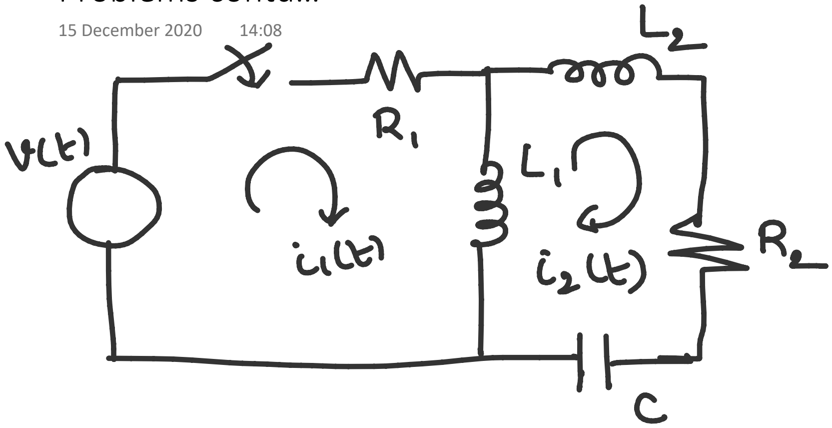
$$L[e^{-t}\sin t] = \frac{b}{(s+a)^2+b^2} = \frac{1}{(s+1)^2+1} \qquad C = 1/2$$

$$= 2/s \Leftarrow \frac{1}{Cs} = \frac{1}{\frac{1}{2}s}$$

$$Z_{eq} =$$

Problems contd...

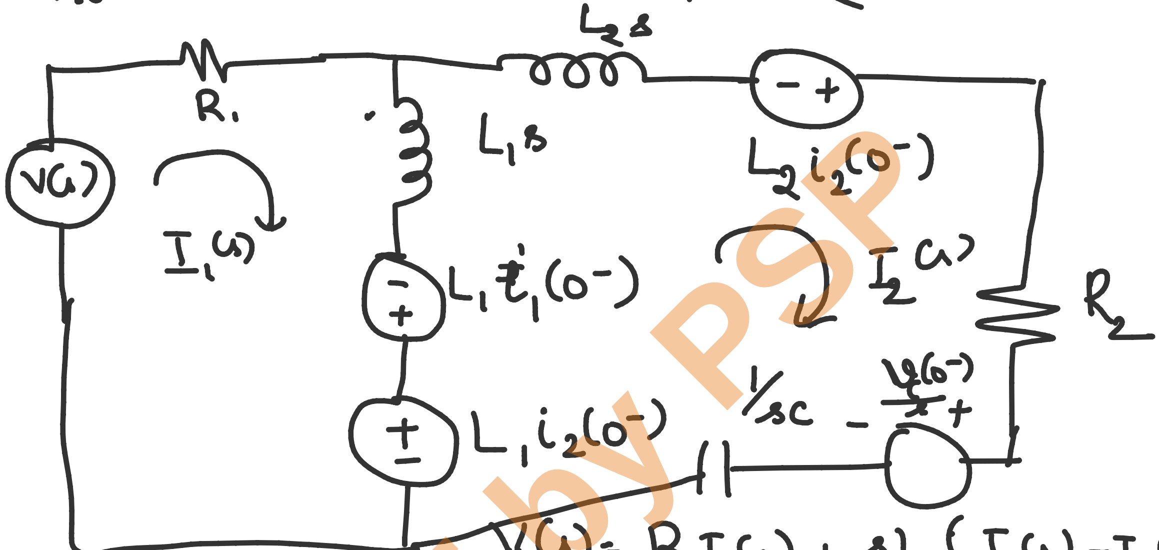
15 December 2020 14:08



$$V(s) = sLI(s) - Li(0^-)$$

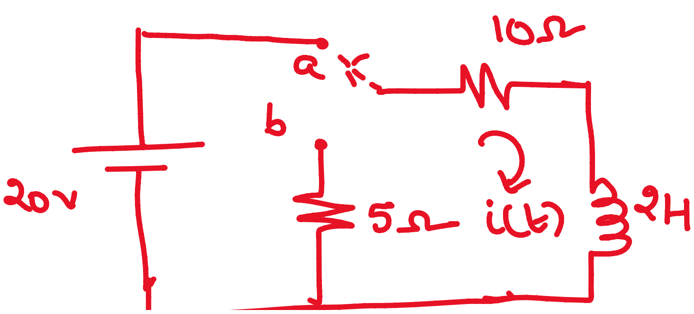
$$V(s) = \frac{I(s)}{sC} + \frac{V_c(0^-)}{s}$$

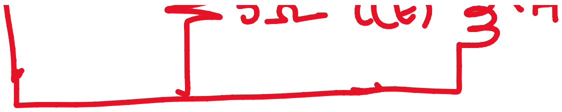
So the current thro' L_1 is $(i_1 - i_2)$



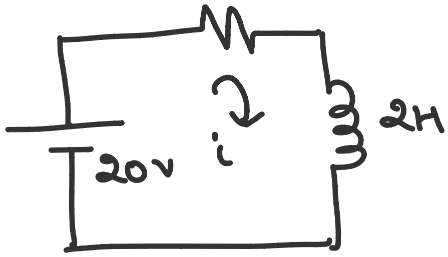
$$V(s) = R I_1(s) + sL_1\left(I_1(s) - I_2(s)\right) - L_1 i_1(0^-) + L_1 i_2(0^-) \quad (1)$$

$$-L_1 i_2(0^-) + L_1 i_1(0^-) + L_1 s\left(I_2(s) - I_1(s)\right) + L_2 s I_2(s) - L_2 i_2(0^-) + R_2 I_2(s) + \frac{V_c(0^-)}{s} + \frac{1}{sC} I_2(s) = 0 \quad (2)$$





Before $t=0$



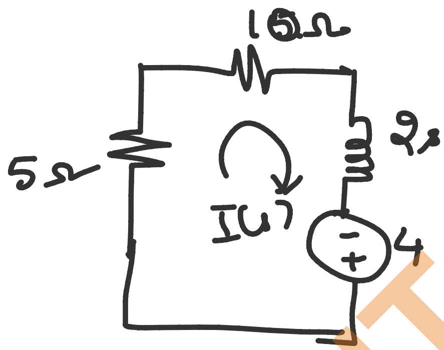
Steady state by the time it is $t=0$.

At s.s. $L \Rightarrow SC$ (acts as a)

$$i = \frac{20}{10} = 2 \text{ Amps}$$

The current thro' inductor owns by $t=0$ is 2Amps.

At $t=0$, the switch is changed to position B



$$V(s) = LsI(s) - Li(0^-)$$

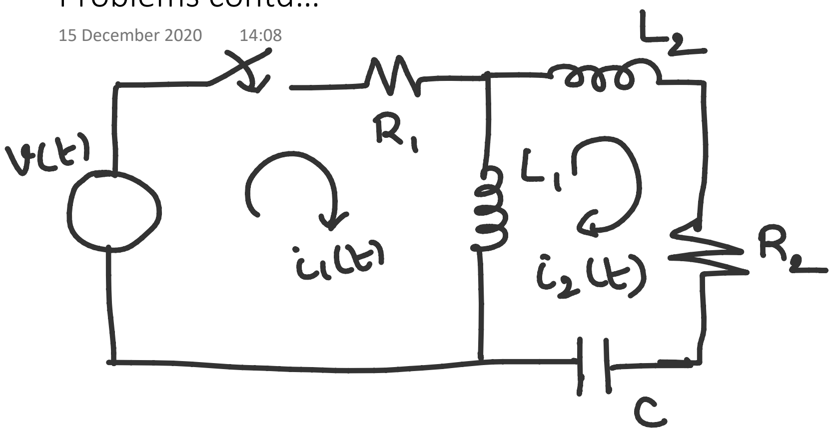
$$= 2sI(s) - 2\times 2$$

$$4 = (15+2s)\,I(s) \qquad I(s) = \frac{4}{2s+15} = \frac{2}{s+7.5}$$

$$i(t) = 2e^{-7.5t} \text{ Amps}$$

Problems contd...

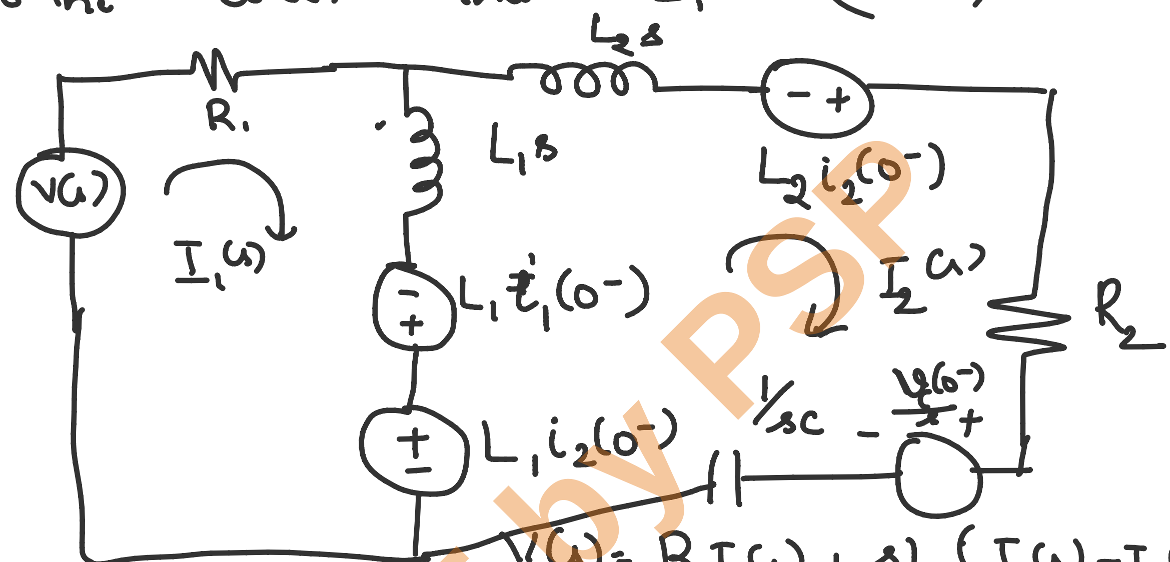
15 December 2020 14:08



$$V(s) = sL I(s) - L i(0^-)$$

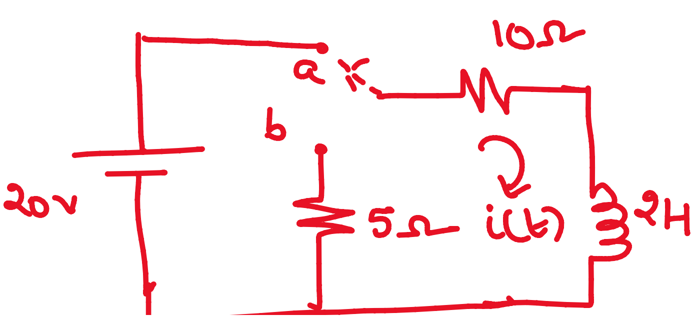
$$V(s) = \frac{I(s)}{sC} + \frac{v_c(0^-)}{s}$$

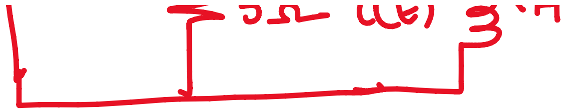
So the current thro' L_1 is $(i_1 - i_2)$



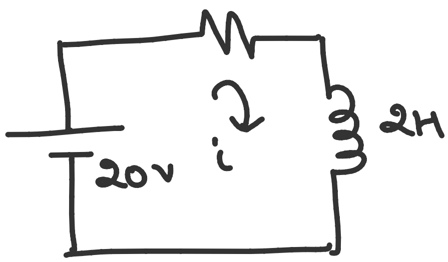
$$V(s) = R I_1(s) + sL_1(I_1(s) - I_2(s)) - L_1 i_1(0^-) + L_1 i_2(0^-) \quad (1)$$

$$-L_1 i_2(0^-) + L_1 i_1(0^-) + L_1 s(I_2(s) - I_1(s)) + L_2 s I_2(s) - L_2 i_2(0^-) + R_2 I_2(s) + \frac{v_c(0^-)}{s} + \frac{1}{sC} I_2(s) = 0 \quad (2)$$





Before $t=0$



Steady state by the time it is $t=0$.

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$$i = \frac{20}{10} = 2 \text{ Amps}$$

The current thro' inductor arms by $t=0$ is 2Amps.

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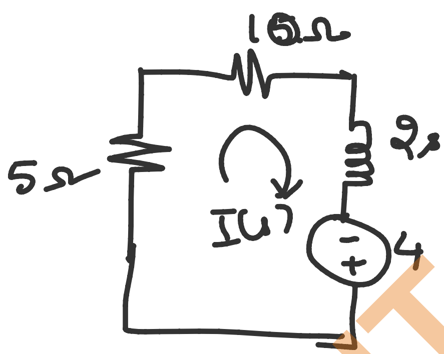


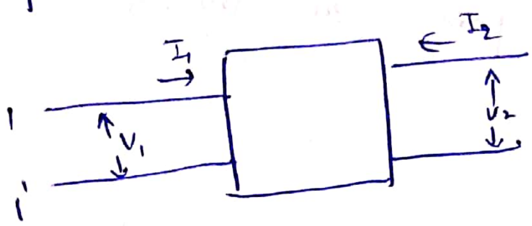
$$V(s) = LsI(s) - Li(0^-)$$
$$= 2sI(s) - 2\times 2$$

$$4 = (15+2s)\,I(s) \qquad I(s) = \frac{4}{2s+15} = \frac{2}{s+7.5}$$

$$i(t) = 2e^{-7.5t} \text{ Amps}$$

Network Function

Network functions give the relation between the transform of the excitation to the transform of the response.



The driving point impedance at port 1-1' is the ratio of the transform voltage at port 1-1' to the transform current at the same port

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

Similarly at port 2-2' is

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

Also the driving point admittances can be defined a

$$Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$$

$$Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

Two Port Network

The four other network functions are called transfer functions.

These functions give the relation between voltage or current at one port to the voltage or current or current voltage at another port as shown.

Voltage Transfer Ratio :

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} \quad , \quad G_{12}(s) = \frac{V_1(s)}{V_2(s)}$$

Current Transfer Ratio

$$\alpha_{12}(s) = \frac{I_1(s)}{I_2(s)} \quad , \quad \alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}$$

Transfer Impedance

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} \quad Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$$

Transfer Admittance

$$Y_{12}(s) = \frac{I_1(s)}{V_2(s)} \quad , \quad Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$$

Pole And Zero

Let $N(s)$, the network function be written as

$$N(s) = \frac{P(s)}{Q(s)} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \dots + b_{m-1} s + b_m}$$

where $a_0, a_1 \ldots a_n$ and $b_0, b_1 \ldots b_m$ are the Coefficients of the polynomials $P(s)$ and $Q(s)$, they are real and positive for a passive network.

If the numerator and denominator of $N(s)$ are factorized, the network function can be written as

$$N(s) = \frac{P(s)}{Q(s)} = \frac{a_0 (s-z_1)(s-z_2) \ldots (s-z_n)}{b_0 (s-p_1)(s-p_2) \ldots (s-p_m)}$$

where $z_1, z_2 \ldots z_n$ are n roots of $P(s)=0$

$p_1, p_2 \ldots p_m$ are m roots of $Q(s)=0$

$\frac{a_0}{b_0}$ - H - scale factor.

$z_1, z_2 \ldots z_n$ are called zeros of the polynomial $N(s)$ and are denoted by 0.

$p_1, p_2 \ldots p_m$ are called poles of the polynomial $N(s)$ and are denoted by $\times$.

The function $N(s)$ becomes zero when s is equal to anyone of the zeros. $N(s)$ becomes infinite when s is equal to anyone of the poles. A network function is completely defined by poles and zeros.

If poles and zeros are not repeated, then the function is said to be simple poles or simple zeros.

If poles and zeros are repeated, then the function is said to be having multiple poles & multiple zeros.

When $n>m$, then $(n-m)$ zeros are at $s=\infty$.

for $m>n$, $(m-n)$ poles are at $s=\infty$.

The network function is said to be stable when the real parts of the zeros or poles are negative. Else, the poles and zeros must lie in the left half of s-plane.

Necessary Conditions for Driving Point Function

The restrictions on pole and zero locations in the driving point function with common factors in $P(s)$ and $Q(s)$ cancelled are listed below.

1. The coefficients in the polynomials $P(s)$ & $Q(s)$ of the network function $N(s) = \frac{P(s)}{Q(s)}$ must be real and positive.

2. Complex or imaginary pole and zeros must occur in conjugate pairs.

3. a. The real part of all poles and zeros must be zero, or negative

b. If the real part is zero, then the poles and zeros must be ~~zero~~. simple.

4. The polynomials P(s) and Q(s) may not have any missing terms between the highest and the lowest degrees, unless all even or all odd terms are missing.

5. The degree of P(s) and Q(s) may differ by zero, or one only.

6. The lowest degree in P(s) and Q(s) may differ in degree by the most one.

Necessary Conditions for Transfer Functions

The restriction on pole and zero location in transfer functions with common factor in P(s) & Q(s) cancelled are listed

1. a. The coefficients in the polynomials P(s) & Q(s) of $N(s) = P(s)/Q(s)$ must be real.

b. The coefficients in Q(s) must be positive, but some of the coefficients in P(s) may be negative.

2. Complex ~~and~~ or imaginary poles and zeros must occur in conjugate pairs.

3. The real parts of poles must be -ve, or zero. If the real part is zero, then the pole must be simple.

4. The polynomial Q(s) may not have any missing terms between the highest and the lowest degree, unless all even or all odd terms are missing.

5. The polynomial P(s) may have missing terms between the lowest and the highest degree.

6. The degree of P(s) may be as small as zero, ~~indeg~~ independent of the degree of Q(s).

7. a. For voltage and current transfer ratios, the maximum degree of P(s) must equal the degree of Q(s).

b. For transfer impedance and transfer admittance, the max. degree of P(s) must equal the degree of Q(s) plus one.

Significance of Poles and Zeros

Poles and zeros are critical frequencies. At poles, the network function become infinite, while at pole zeros, the network function become zero.

At other complex frequencies, the network functions has a finite non-zero value.

Poles and zeros provide useful information in network functions. Consider the following cases.

ⓐ Driving Point Impedance

$$Z(s) = \frac{V(s)}{I(s)}$$

A pole of $Z(s)$ implies zero current for a finite voltage which means an open circuit. A zero of $Z(s)$ implies no voltage for a finite current or a short circuit.

ⓑ Driving point Admittance

$$Y(s) = \frac{I(s)}{V(s)}$$

A pole of $Y(s)$ implies zero voltage for a finite current which means a short circuit. A zero of $Y(s)$ implies zero current for a finite value of voltage which gives an open circuit

(c) Voltage Transfer Ratio

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} \Rightarrow V_2(s) = G_{21}(s) V_1(s)$$

To obtain output voltage, we require the product of input and transfer function.

By using partial fractions, we can obtain a pole of $G_{21}(s)$ or $V_1(s)$

$$G_{21}(s) V_1(s) = \sum_{i=1}^{n} \frac{A}{s-a_i} + \sum_{j=1}^{m} \frac{A}{s-a_j}$$

where n and m are the no. of poles of $G_{21}(s)$ and $V_1(s)$ respectively.

The frequencies a_i from the natural complex ~~function~~ frequencies corresponding to free oscillation and depend on the network function $G_{21}(s)$, where as the frequencies a_j constitute the complex frequencies corresponding to forced oscillations and depend on the driving force $V_1(s)$.

From the above discussion, it can be observed that the poles determine the time variation of the response where as the zeros determine the magnitude response.

(d) Other network function

Significance of poles and zeros in other transfer functions is the same as discussed above.

In each of the case, poles determine the time domain behaviour and zeros determine the magnitude of each of the term of the response.

Properties of Driving Point Functions

a. The driving point functions is a ratio of polynomial is s. Polynomials are obtained from the transform impedances of the elements and their combinations.

$$\left.\begin{aligned} P(s) &= a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n \\ Q(s) &= b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m \end{aligned}\right\} - ①$$

are the numerator & denominator polynomials respectively.

$$P(s) = (s - z_1)(s - z_2) \dots (s - z_n)$$

$$Q(s) = (s - p_1)(s - p_2) \dots (s - p_m)$$

$$\Rightarrow N(s) = \frac{P(s)}{Q(s)}$$

$z_1, z_2 \dots z_n$ are called zeros of $N(s)$. $p_1, p_2 \dots p_m$ are called poles of $N(s)$ $[N(p_1) = N(p_2) = \dots N(p_m) = \infty]$

b) i. $N(s)$ is a driving point impedance ⓘ $Z(s)$

$N(s) = Z(s) = \frac{V(s)}{I(s)}$, A zero of $N(s)$ is a zero of $V(s)$ & a pole of $N(s)$ is a zero of $I(s)$. poles of $Z(s)$ are those freq. corresponding to open ckt

ii. $N(s) = Y(s) = \frac{I(s)}{V(s)}$, A zero of $Y(s)$ is a zero of $I(s)$ & pole of $Y(s)$ is a zero of $V(s)$.

c. Since all the elements in the circuit are real positive quantities the coefficients $a_0, a_1, a_2 \dots a_n$ & $b_0, b_1 \dots b_m$ are real and positive. Therefore, any zeros and poles, if complex, must occur in conjugate pairs.

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d. The real parts of all zeros and poles must be negative or zero. Using partial fractions, we know that this gives rise to a term of a form $\frac{A}{s-p}$ where inverse will be e^{pt}. The real part of e^{pt} tends to zero as t tends to infinity if p is a -ve quantity.

e. poles & zeros lying on jω axis must be simple. If not the time response will be of the form $t^k e^{j\omega t}$ which tends to infinity as t tends to infinity. So the fn becomes unstable.

f. The degree of P(s) and Q(s) may differ by zero or one only.

At very high freq. the term $a_0 s^n$ dominates over the other terms in the numerator & the term $b_0 s^m$ dominates over other terms in the denominator.

$$\lim_{s\to\infty} N(s) = \lim_{s\to\infty} \frac{a_0}{b_0} s^{n-m}$$

Consider R, L, C & M

If $n=m$, the function $R = \frac{a_0}{b_0}$

$n=m+1$, the function is Ls.

$m=n+1$, the function is like $\frac{1}{Cs}$. $||^{ly}$ for admittance

g. The lowest degree in P(s) & Q(s) may differ by zero or one only.

As $s \to 0$, the higher power of s tends to 0 faster than s.

So $N(s) = \dfrac{a_{n-1}s + a_n}{b_{m-1}s + b_m}$

$N(s)$ is of the form K_1, $K_2 s$ or $\frac{K_3}{s}$. Hence $P(s)$ & $Q(s)$ can differ at most in one degree.

h. $P(s)$ and $Q(s)$ cannot have missing terms unless all even or all odd degree terms are absent.

An RL, RC and RLC may have form $(as+b)$, $(a+b/s)$ and (as^2+bs+c). If it is a combination of L & C then it is of the form (as^2+b). If something of this form is multiplied by s, then the resulting function contains only odd powers of s.

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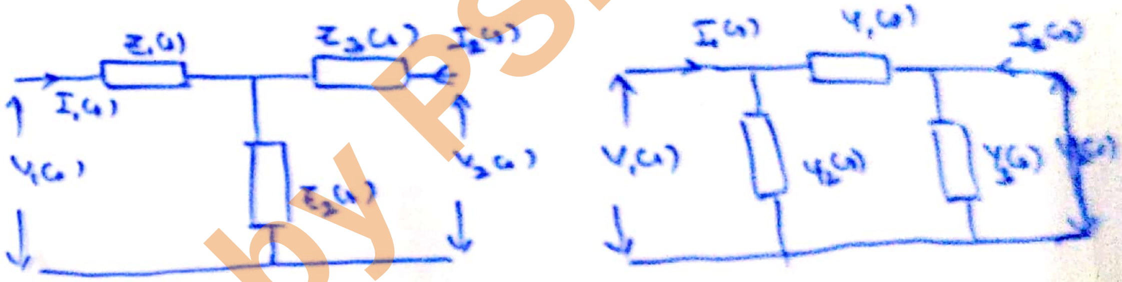
Properties of Transfer Functions

a. The transfer function is a ratio of polynomials.

b. For stability, the poles must be real & -ve or complex conjugate pairs.

c. There are no restrictions on the zeros of the transfer function, $P(s)$ can have missing terms. Also coefficients of powers of s in $P(s)$ can be -ve.

d. For $G(s)$ and $\alpha(s)$, the degree of the numerator polynomial $P(s)$ is less than or equal to the degree of $Q(s)$



$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)} \quad - \quad (1)$$

$$\alpha(s) = \frac{I_2(s)}{I_1(s)} = \frac{Y_2(s)}{Y_1(s)+Y_2(s)} \quad - \quad (2)$$

$Z_1(s), Z_2(s)$ & $Z_3(s)$, $Y_1(s), Y_2(s)$ & $Y_3(s)$ can be thought of as the driving point functions of some one port; they have to satisfy the properties of driving point immittance functions.

$$Z_1(s) = K \frac{(s+\alpha_1)(s+\alpha_2)\cdots(s+\alpha_{n_1})}{(s+\beta_1)(s+\beta_2)\cdots(s+\beta_{m_1})}$$

$$Z_2(s) = K_2 \frac{(s+\gamma_1)(s+\gamma_2)\cdots(s+\gamma_{n_2})}{(s+\delta_1)(s+\delta_2)\cdots(s+\delta_{m_2})}$$

Then the degree of $P(s) = n_1+m$ & degree of $Q(s)$ $= n_1+m_2$ or n_2+m_1, which ever is greater

If $n_1+m_2 > n_2+m_1$, the degree of $P(s)$ equals the 6.
degree of $Q(s)$.

If $n_1+m_2 < n_2+m_1$, degree of $Q(s) = n_2+m_1$ and degree of $P(s)$ is less than degree of $Q(s)$

Similarly, assuming $Y_1(s)$ & $Y_2(s)$ as ratios of polynomials and substituting these expressions in $Z(s)$, it can be shown that the degree of the numerator of $Z(s)$ is less than or equal to the degree of denominator.